Online Appendix

A Bargaining-Based Model of Security Design

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This appendix contains an extension of the model to a setting with more general contracts, and a proof of existence of bilaterally stable prices.

1 More General Contracts

The analysis focused on joint and separate contracts, but more general contracts can be written if securities depend on the vector of agreements. Formally, a security $S : \mathbb{R}^N \to \mathbb{R}_+$ indicates the payment to the investors as a function of the vector of agreements with the buyers. Following the agreements $(x_1, \ldots, x_N)$, the investors will receive $S(x_1, \ldots, x_N)$ and the firm will keep the remainder $X - S(x_1, \ldots, x_N)$, where $X = \sum_{i=1}^{N} x_i$. While joint and separate contracts are obviously special cases, we will show that financing each project with debt separately remains optimal within a large class of these securities.

Vectors are denoted by bold symbols. Given a price vector $\boldsymbol{x} = (x_1, \ldots, x_N)$, let $\boldsymbol{\hat{x}_j} = (x_1 \ldots, x_{j-1}, 0, x_{j+1}, \ldots, x_N)$ denote the vector that results when replacing the j-th element of $\boldsymbol{x}$ with 0, and the *marginal payment* to the investors from reaching an agreement with buyer j is therefore $S(\boldsymbol{x}) - S(\boldsymbol{\hat{x}_j})$. The key property is that the sum of the marginal payments to the investors does not exceed the total payment:

$$\sum_{i=1}^{N} S(\boldsymbol{x}) - S(\boldsymbol{\hat{x}_i}) \leq S(\boldsymbol{x}) \text{ for all } \boldsymbol{x} \geq 0 \tag{1.1}$$

Proposition 1. *Financing each project with debt separately is optimal within the class of securities satisfying (1.1).*

The proof is identical to that of Proposition 3.

Proof. First, if the prices $x_1, \ldots, x_N$ are bilaterally stable, then the firm's gain from each trade will not exceed the buyer's gain:

$$x_i - (S(\boldsymbol{x}) - S(\hat{\boldsymbol{x}}_i)) \leq v_i - x_i, \forall i$$

Second, if the security satisfies (1.1), the firm's profit will not exceed the sum of the firm's gains from each trade:

$$X - S(\boldsymbol{x}) \leq \sum_{i=1}^{N} x_i - (S(\boldsymbol{x}) - S(\hat{\boldsymbol{x}}_i)), \text{ where } X = \sum_{i=1}^{N} x_i$$

Thus, the firm's payoff will not exceed the buyers', $X - S(\boldsymbol{x}) < V - X$, and half the social surplus is an upper bound. □

These securities are more general but they also require stronger, and perhaps unrealistic, assumptions on the contracts. Observe that if the firm can shuffle the proceeds from one project to another, then the payment to the investors will only depend on the sum of the proceeds. We therefore focused our analysis on the more common contracts.

2 Existence of Bilaterally Stable Outcome

Lemma 6. If S is a smooth and concave function, then a bilaterally stable outcome exists.

Proof. Assume without the loss of generality that $S'(Y) < 1$ for all $Y \geq 0$ (if $S'_+(0) \geq 1$, then $(0, \ldots, 0)$ is bilaterally stable). Recall in a bilateral bargaining game with payoffs $G_j(S, Y)$ and continuation probability $p < 1$, the maximal SPE price when the buyer (resp. the firm) makes the first offer is $x_j^B(Y, p)$ (resp. $x_j^F(Y, p)$). The functions are well defined (see Lemma 4). We will first show that $x_j^F(Y, p)$ and $x_j^B(Y, p)$ converge, as $p \to 1$, to

$$x_j(Y) \quad := \quad \max\left\{x : x + \frac{x - S(Y+x) + S(Y)}{1 - S'(Y+x)} = v_j \text{ and } x \leq v_j\right\} \tag{2.1}$$

Since $S' < 1$, $x_j(Y)$ exists and $v_j > x_j(Y) > 0$. It follows from Lemma 4 that there is a pair of prices x_B and x_F, where $v_j > x_F > x_B > 0$, that solves

$$v_j - x_F \quad = \quad p(v_j - x_B) \tag{2.2}$$

$$Y + x_B - S(Y + x_B) \quad = \quad (1-p)\,(Y - S(Y)) + p\,(Y + x_F - S(Y + x_F)) \tag{2.3}$$

and the prices $x_j^B(Y,p)$ and $x_j^F(Y,p)$ are the maximal prices that solve (2.2) and (2.3). Therefore, if we let

$$g(x,Y,p) = x - S(Y+x) + S(Y) - p\,(px + (1-p)v_j - S(Y + px + (1-p)v) + S(Y))$$

then

$$x_j^B(Y,p) = \max\{x : \; g(x,Y,p) = 0 \text{ and } x \leq v_j\} \tag{2.4}$$

By the intermediate value theorem,

$$g(x,Y,p) = 0 \iff x + \frac{x - S(Y+x) + S(Y)}{p\,(1 - S'(Y + \tilde{x}_p))} = v_j, \tag{2.5}$$

where $x < \tilde{x}_p < px + (1-p)v_j$. Hence, given $\delta > 0$, we have that $x_j^B(Y,p) \in (x_j(Y) - \delta, x_j(Y) + \delta)$, for all p sufficiently large, and $x_j^B(Y,p) \to x_j(Y)$ as $p \to 1$. Moreover, $x_j(Y)$ is continuous by the maximum theorem. Finally, prices $y_1, \ldots, y_N$ that satisfy $y_j = x_j(Y_{-j})$, $\forall j$, are bilaterally stable. Therefore, define $\phi : [0, v_1] \times \ldots \times [0, v_N] \to [0, v_1] \times \ldots \times [0, v_N]$ such that $\phi(y_1, \ldots, y_N) = (x_1(Y_{-1}), \ldots, x_N(Y_{-N}))$. Since ϕ is a continuous function from a convex and compact set onto itself, there exists a fixed point that is bilaterally stable. $\square$