Online Appendix
Paths to the Frontier

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OA Proofs for Section I

Proof of Lemma 1. Before we begin, as a matter of notation, when we consider the concatenation $h_t \sqcup h_\tau = (\mathbf{x}^s, \mathbf{d}^s)_{s<t+\tau}$ of two histories h_t and h_τ, with $h_\tau = (\tilde{\mathbf{x}}^s, \tilde{\mathbf{d}}^s)_{s<\tau}$, then for all $s = 0, ..., \tau - 1$ we take $(\mathbf{x}^{t+s}, \mathbf{d}^{t+s})$ to be $(P^{-1}_{\mathbf{z}^t}(\tilde{\mathbf{x}}^s), \tilde{\mathbf{d}}^s)$, where $\mathbf{z}^t$ is the status quo at time t under the history h_t.

Fix an SPE σ and a history h_t with status quo $\mathbf{z} = \mathbf{z}(h_t)$. Consider strategy profile $\hat{\sigma}$ such that, for $i = 1, 2$ and for each history h_τ, $\hat{\sigma}_i(h_\tau) = \sigma_i(h_t \sqcup h_\tau)$. Assumption 1 guarantees that $\hat{\sigma}$ is a SPE of the game. We now show that, for $i = 1, 2$,

$$V_i^\sigma(h_t) = z_i + (1 - z_1 - z_2)V_i^{\hat{\sigma}}(h_0).$$

Suppose for a contradiction that the result is not true. Then there exists $\epsilon > 0$ and $j \in \{1, 2\}$ such that $|V_i^\sigma(h_t) - z_j - (1 - z_1 - z_2)V_j^{\hat{\sigma}}(h_0)| > \epsilon$. Pick T such that $(1 - \delta)\delta^T < \epsilon/4$. Consider strategy profiles σ^T and $\hat{\sigma}^T$ such that: (a) for all histories h_s with $s \leq T$, $\sigma^T(h_t \sqcup h_s) = \sigma(h_t \sqcup h_s)$ and $\hat{\sigma}^T(h_s) = \hat{\sigma}(h_s)$, and (b) for all histories h_s with $s > T$, both players reject all proposals at history $h_t \sqcup h_s$ under σ^T, and both players reject all proposals at history h_s under $\hat{\sigma}^T$.[1]

Since $(1 - \delta)\delta^T < \epsilon/4$, for $i = 1, 2$ we have $|V_i^\sigma(h_t) - V_i^{\sigma^T}(h_t)| < \epsilon/4$ and $|V_i^{\hat{\sigma}}(h_0) - V_i^{\hat{\sigma}^T}(h_0)| < \epsilon/4$. Therefore, since $|V_j^\sigma(h_t) - z_j - (1 - z_1 - z_2)V_j^{\hat{\sigma}}(h_0)| > \epsilon$, we have

$$|V_j^{\sigma^T}(h_t) - z_j - (1 - z_1 - z_2)V_j^{\hat{\sigma}^T}(h_0)| > \epsilon/2.$$

For each history h_T of length T, let $(V_i^{\hat{\sigma}^T}(h_T))_{i=1,2}$ (resp., $(V_i^{\sigma^T}(h_t \sqcup h_T))_{i=1,2}$) denote players' continuation payoffs at history h_T under $\hat{\sigma}^T$ (resp., at history $h_t \sqcup h_T$ under σ^T). Let $\mathbf{z}(h_T)$ denote the status quo under history h_T, and $\mathbf{z}(h_t \sqcup h_T) = \mathbf{z} + (1 - z_1 - z_2)\mathbf{z}(h_T)$ the status quo under history $h_t \sqcup h_T$. Note that:

$$\begin{aligned} V_i^{\hat{\sigma}^T}(h_T) = &\operatorname{prob}_{\mathbf{z}(h_T)}(\mathbf{x} \in A^{\hat{\sigma}}(h_T))\mathbb{E}_{\mathbf{z}(h_T)}[x_i | \mathbf{x} \in A^{\hat{\sigma}}(h_T)] \\ &+ (1 - \operatorname{prob}_{\mathbf{z}(h_T)}(\mathbf{x} \in A^{\hat{\sigma}}(h_T)))z_i(h_T), \end{aligned}$$

where $A^{\hat{\sigma}}(h_T)$ is the set of policies that both players accept under $\hat{\sigma}$, and where the equality follows since policy doesn't change after time T under $\hat{\sigma}^T$. Similarly,

$$\begin{aligned} V_i^{\sigma^T}(h_t \sqcup h_T) &= \operatorname{prob}_{\mathbf{z}(h_t \sqcup h_T)}(\mathbf{x} \in A^\sigma(h_t \sqcup h_T))\mathbb{E}_{\mathbf{z}(h_t \sqcup h_T)}[x_i | \mathbf{x} \in A^\sigma(h_\tau \sqcup h_T)] \\ &\quad + (1 - \operatorname{prob}_{\mathbf{z}(h_\tau \sqcup h_T)}(\mathbf{x} \in A^\sigma(h_\tau \sqcup h_T)))z_i(h_\tau \sqcup h_T) \\ &= \operatorname{prob}_{\mathbf{z}(h_T)}(\mathbf{x} \in A^\sigma(h_T))\mathbb{E}_{\mathbf{z}(h_T)}[z_i + (1 - z_1 - z_2)x_i | \mathbf{x} \in A^\sigma(h_T)] \\ &\quad + (1 - \operatorname{prob}_{\mathbf{z}(h_T)}(\mathbf{x} \in A^\sigma(h_T)))(z_i + (1 - z_1 - z_2)z_i(h_T)) \\ &= z_i + (1 - z_1 - z_2)V_i^{\hat{\sigma}^T}(h_T), \end{aligned}$$

where the second equality uses Assumption 1.

[1]We stress that σ^T and $\hat{\sigma}^T$ need not be equilibria of the game.

Suppose that $V_i^{\hat{\sigma}^T}(h_t \sqcup h_s) = z_i + (1 - z_1 - z_2)V_i^{\sigma^T}(h_s)$ for histories h_s of length $s = \tau + 1, ..., T$. Consider a history h_τ of length τ. Let $\mathbf{z}(h_\tau)$ be the status quo under h_τ, and $\mathbf{z}(h_t \sqcup h_\tau) = \mathbf{z} + (1 - z_1 - z_2)\mathbf{z}(h_\tau)$ the status quo under $h_t \sqcup h_\tau$. For each $\mathbf{x} \in X$, let $h_{\tau+1}^{\mathbf{x}}$ denote the history of length $\tau + 1$ that follows h_τ if policy $\mathbf{x}$ is implemented at time t. Then

$$
\begin{aligned}
V_i^{\hat{\sigma}^T}(h_\tau) = \text{prob}_{\mathbf{z}(h_\tau)}(\mathbf{x} \in A^{\hat{\sigma}}(h_\tau))\mathbb{E}_{\mathbf{z}(h_\tau)}[(1-\delta)x_i + \delta V_i^{\hat{\sigma}^T}(h_{\tau+1}^{\mathbf{x}}) | \mathbf{x} \in A^{\hat{\sigma}}(h_\tau)] \\
+ (1 - \text{prob}_{\mathbf{z}(h_\tau)}(\mathbf{x} \in A^{\hat{\sigma}}(h_\tau)))((1-\delta)z_i(h_\tau) + \delta V_i^{\hat{\sigma}^T}(h_{\tau+1}^{\mathbf{z}(h_\tau)}))
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
V_i^{\sigma^T}(h_t \sqcup h_\tau) &= \text{prob}_{\mathbf{z}(h_t \sqcup h_\tau)}(\mathbf{x} \in A^{\sigma}(h_t \sqcup h_\tau)) \times \\
&\quad \times \mathbb{E}_{\mathbf{z}(h_t \sqcup h_\tau)}[(1-\delta)x_i + \delta V_i^{\sigma^T}(h_t \sqcup h_{\tau+1}^{\mathbf{x}}) | \mathbf{x} \in A^{\sigma}(h_t \sqcup h_\tau)] \\
&\quad + (1 - \text{prob}_{\mathbf{z}(h_t \sqcup h_\tau)}(\mathbf{x} \in A^{\sigma}(h_t \sqcup h_\tau)))((1-\delta)z_i(h_t \sqcup h_\tau) + \delta V_i^{\sigma^T}(h_t \sqcup h_{\tau+1}^{\mathbf{z}(h_t \sqcup h_\tau)})) \\
&= \text{prob}_{\mathbf{z}(h_\tau)}(\mathbf{x} \in A^{\hat{\sigma}}(h_\tau)) \times \\
&\quad \times \mathbb{E}_{\mathbf{z}(h_t)}[z_i + (1 - z_1 - z_2)((1-\delta)x_i + \delta V_i^{\hat{\sigma}^T}(h_{t+1}^{\mathbf{x}})) | \mathbf{x} \in A^{\hat{\sigma}}(h_\tau)] \\
&\quad + (1 - \text{prob}_{\mathbf{z}(h_\tau)}(\mathbf{x} \in A^{\hat{\sigma}}(h_\tau)) \times \\
&\quad \times (z_i + (1 - z_1 - z_2)((1-\delta)z_i(h_\tau) + \delta V_i^{\hat{\sigma}^T}(h_{\tau+1}^{\mathbf{z}(h_\tau)}))) = z_i + (1 - z_1 - z_2)V_i^{\hat{\sigma}^T}(h_\tau).
\end{aligned}
$$

Hence, $V_i^{\sigma^T}(h_t) - z_i - (1 - z_1 - z_2)V_i^{\hat{\sigma}^T}(h_0) = 0$, a contradiction.

In the case of RME, the same contradiction follows if we take $\hat{\sigma} = \sigma$. ■

OB Proofs for Section III

Proof of Lemma 3. We start with part (i). For each $\mathbf{z} \in X$, $\mathbb{E}_{\mathbf{z}}[\cdot]$ is the expectation operator under distribution $F_{\mathbf{z}}$. Let $\mathbb{E}[\cdot]$ be the expectation operator under distribution $F_{\mathbf{0}} = F$. We prove the result by induction.

Consider a subgame starting at period $t = T$ with status quo $\mathbf{z}^T = \mathbf{z} \in X$. Note that

$$V_i(\mathbf{z}, T; T) = \mathbb{E}_{\mathbf{z}}[x_i] = z_i + (1 - z_1 - z_2)\mathbb{E}[x_i],$$

where the first equality follows since, at time T both players accept any policy, and the second equality follows from Assumption 1.

Now, consider the game with deadline $T = 0$. Player i's equilibrium payoffs satisfy $W_i(0) = \mathbb{E}[x_i]$. Hence,

$$V_i(\mathbf{z}, T; T) = z_i + (1 - z_i - z_j)W_i(0)$$

which establishes the basis case.

For the induction step, suppose that (5) holds for all t such that $T - t = 0, 1, ..., n-1$ and for all $\mathbf{z} \in X$. Fix a subgame starting at period $\tilde{t}$ with $T - \tilde{t} = n$ and with status quo

$\mathbf{z}^{\tilde{t}} = \mathbf{z} \in X$. We abuse previous notation and in this proof let $A_{\mathbf{z}}(\tilde{t})$ be the set of policies that both players accept at period $\tilde{t}$ when $\mathbf{z}^{\tilde{t}} = \mathbf{z}$; that is,

$$\begin{aligned} A_{\mathbf{z}}(\tilde{t}) &= \left\{ \mathbf{x} \in X(\mathbf{z}) : (1-\delta)x_i + \delta V_i(\mathbf{x}, \tilde{t}+1; T) \geq (1-\delta)z_i + \delta V_i(\mathbf{z}, \tilde{t}+1; T) \text{ for } i = 1, 2 \right\} \\ &= \left\{ \mathbf{x} \in X(\mathbf{z}) : (x_i - z_i) \geq (x_1 + x_2 - z_1 + z_2)\delta W_i(T - \tilde{t} - 1) \text{ for } i = 1, 2 \right\}, \end{aligned}$$

where the second line follows since, by the induction hypothesis, (5) holds for $t = \tilde{t}+1$. Note then that

$$\begin{aligned} V_i(\mathbf{z}, \tilde{t}; T) = {} & \text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))\mathbb{E}_{\mathbf{z}} \left[(1-\delta)x_i + \delta V_i(\mathbf{x}, \tilde{t}+1; T) \,\middle|\, \mathbf{x} \in A_{\mathbf{z}}(\tilde{t}) \right] \\ & + \text{prob}(\mathbf{x} \notin A_{\mathbf{z}}(\tilde{t})) \left((1-\delta)z_i + \delta V_i(\mathbf{z}, \tilde{t}+1; T) \right) \\ = {} & \text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))\mathbb{E}_{\mathbf{z}} \left[x_i + (1 - x_1 - x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{z} \in A_{\mathbf{z}}(\tilde{t}) \right] \\ & + \text{prob}(\mathbf{x} \notin A_{\mathbf{z}}(\tilde{t})) \left(z_i + (1 - z_1 - z_2)\delta W_i(T - \tilde{t} - 1) \right) \\ = {} & \text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))\mathbb{E}_{\mathbf{z}} \left[(x_i - z_i) + (z_1 + z_2 - x_1 - x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{x} \in A_{\mathbf{z}}(\tilde{t}) \right] \\ & + z_i + (1 - z_1 - z_2)\delta W_i(T - \tilde{t} - 1) \end{aligned} \tag{O1}$$

where the second equality follows since, by the induction hypothesis, (5) holds for $t = \tilde{t}+1$, and the last inequality follows since $\text{prob}(\mathbf{x} \notin A_{\mathbf{z}}(\tilde{t})) = 1 - \text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))$.

Consider next a game with deadline $T - \tilde{t}$. Let $\tilde{A}$ be the set of policies that both players accept at the first period of the game:

$$\begin{aligned} \tilde{A} &= \left\{ \mathbf{x} \in X : (1-\delta)x_i + \delta V_i(\mathbf{x}, 1; T - \tilde{t}) \geq \delta V_i(\mathbf{0}, 1; T - \tilde{t}) \text{ for } i = 1, 2 \right\} \\ &= \left\{ \mathbf{x} \in X : x_i \geq (x_1 + x_2)\delta W_i(T - \tilde{t} - 1) \text{ for } i = 1, 2 \right\}, \end{aligned}$$

where the second line follows since, by the induction hypothesis, for all $V_i(\mathbf{x}, 1; T - \tilde{t}) = x_i + (1 - x_i - x_j)W_i(T - \tilde{t})$ for all $\mathbf{x}$. Player i's payoff in this game is equal to

$$\begin{aligned} W_i(T - \tilde{t}) &= \text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[(1-\delta)x_i + \delta V_i(\mathbf{x}, 1; T - \tilde{t}) \,\middle|\, \mathbf{x} \in \tilde{A} \right] + \text{prob}(\mathbf{x} \notin \tilde{A})\delta V_i(\mathbf{0}, 1; T - \tilde{t}) \\ &= \text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[x_i - (x_1 + x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{x} \in \tilde{A} \right] + \delta W_i(T - \tilde{t} - 1) \end{aligned} \tag{O2}$$

Assumption 1 implies that

$$\begin{aligned} & \text{prob}(\mathbf{x} \in A_{\mathbf{z}}(\tilde{t}))\mathbb{E}_{\mathbf{z}} \left[x_i - z_i + (z_1 + z_2 - x_1 - x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{x} \in A_{\mathbf{z}}(\tilde{t}) \right] \\ = & (1 - z_1 - z_2)\text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[x_i - (x_1 + x_2)\delta W_i(T - \tilde{t} - 1) \,\middle|\, \mathbf{x} \in \tilde{A} \right]. \end{aligned}$$

Combining this with (O1) and (O2),

$$V_i(\mathbf{z}, \tilde{t}; T) = z_i + (1 - z_1 - z_2)W_i(T - \tilde{t}).$$

which establishes the result.

Now let us turn to part (ii). The proof is again by induction. Consider the game with deadline $T = 0$. Since it is optimal for both players to accept any alternative $\mathbf{x} \in X$ that is drawn, player i's payoff in this game satisfies $W_i(T) = \mathbb{E}[x_i] = \Phi_i(\mathbf{0})$. Suppose next that $W_i(\tau) = \Phi_i^{\tau+1}(\mathbf{0})$ for all $\tau = 0, ..., T-1$, and consider game with deadline T. The set of alternatives that both players accept in the initial period are given by

$$\begin{aligned}\tilde{A} &= \{\mathbf{x} \in X : (1-\delta)x_i + \delta V_i(\mathbf{x}, 1; T) \geq \delta V_i(\mathbf{0}, 1; T) \text{ for } i = 1, 2\} \\ &= \{\mathbf{x} \in X : x_i \geq (x_1 + x_2)\delta W_i(T-1) \text{ for } i = 1, 2\},\end{aligned}$$

where the second line follows from part (i). Player i's payoff $W_i(T)$ satisfies

$$\begin{aligned}W_i(T) &= \text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[(1-\delta)x_i + \delta V_i(\mathbf{x}, 1; T) \middle| \mathbf{x} \in \tilde{A}\right] + \text{prob}(\mathbf{x} \notin \tilde{A})\delta V_i(\mathbf{0}, 1; T) \\ &= \text{prob}(\mathbf{x} \in \tilde{A})\mathbb{E}\left[x_i - (x_1 + x_2)\delta W_i(T-1) \middle| \mathbf{x} \in \tilde{A}\right] + \delta W_i(T-1) \qquad \text{(O3)}\end{aligned}$$

where the equality follows after using part (i). By the induction hypothesis, $\mathbf{W}(T-1) = \Phi^T(\mathbf{0})$, and so $\tilde{A} = A(\Phi^T(\mathbf{0}))$. Using this in (O3), $W_i(T) = \Phi(\Phi^T(\mathbf{0})) = \Phi^{T+1}(\mathbf{0})$. ■

Proof of Proposition 5. We start with part (i) and recall various facts from the proof of Proposition 2. First, recall that $\underline{V}^\delta$ is the smaller of the two solutions to the quadratic equation $\frac{1}{3}\underline{g}\frac{(1-\delta\underline{V}^\delta)^2}{1-\delta} = \underline{V}^\delta$, where $\underline{g} \in (0, \underline{f})$.[2] Also from the proof of Proposition 2, for $i, j = 1, 2$, $i \neq j$, $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i}$ is given by (A3) and lies in the interval $[\delta - \frac{\overline{f}}{3}\delta(1 - \delta(W_1 + W_2)), \delta]$ while $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j}$ is given by (A4) and lies in $[-\frac{\overline{f}}{3}\delta(1 - \delta(W_1 + W_2)), 0]$. The proof of Proposition 2 also showed that for all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i} \geq 0 \geq \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j}$. Finally, it showed that for all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, $\Phi^\delta(\mathbf{W}) \in Y^\delta$.

Now fix $\delta > \underline{\delta}$. Towards establishing the result, we first show that if $\mathbf{W} \in Y^\delta$, then $(\Phi^\delta)^T(\mathbf{W})$ converges to a fixed point of Φ as $T \to \infty$. To see why, fix $\mathbf{W}^0 \in Y^\delta$, and let $\{\mathbf{W}^t\}_{t=0}^\infty$ be such that, for $t = 1, 2, ...$, $\mathbf{W}^t = (\Phi^\delta)^t(\mathbf{W}) = (\Phi^\delta)(\mathbf{W}^{t-1})$. Note then that $\mathbf{W}^t \in Y^\delta$ for all t.[3]

There are two cases to consider: (a) there exists $s \geq 1$ and $i = 1, 2, i \neq j$ such that $W_i^s \geq W_i^{s-1}$ and $W_j^s \leq W_j^{s-1}$, and (b) for all $s \geq 1$, either $W_1^s \geq W_1^{s-1}$ and $W_2^s \geq W_2^{s-1}$ or $W_1^s \leq W_1^{s-1}$ and $W_2^s \leq W_2^{s-1}$.

Consider first case (a), so there exists $s \geq 1$ and $i = 1, 2, i \neq j$ such that $W_i^s \geq W_i^{s-1}$ and $W_j^s \leq W_j^{s-1}$. Since $\Phi_i^\delta(W_i, W_j)$ is increasing in W_i and decreasing in W_j whenever $\mathbf{W} \in Y^\delta$, it follows that $W_i^{s+1} = \Phi_i(\mathbf{W}^s) \geq \Phi_i(\mathbf{W}^{s-1}) = W_i^s$ and $W_j^{s+1} = \Phi_j(\mathbf{W}^s) \leq \Phi_j(\mathbf{W}^{s-1}) = W_j^s$. Applying the same argument inductively, we get that $\{W_i^t\}$ is an increasing sequence and $\{W_j^t\}$ is a decreasing sequence for all $t \geq s$. Since $\mathbf{W}^t \in X$ for all t, $\mathbf{W}^t$ converges to some $\mathbf{W}^*$ as $t \to \infty$.

Consider next case (b). For $i, j = 1, 2, j \neq i$, define

$$M_{i,i} := \sup_{\mathbf{W} \in Y^\delta} \left| \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i} \right| \qquad M_{i,j} := \sup_{\mathbf{W} \in Y^\delta} \left| \frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j} \right|$$

[2] For this proof, we don't need Assumption 3 to hold, and we also don't need $\underline{g} > \frac{3}{4}\gamma$.

[3] Indeed, for all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, $\Phi^\delta(\mathbf{W}) \in Y^\delta$.

Note that, for $\delta > \underline{\delta}$, we have that $M_{i,i} \in [0, \delta]$ and $M_{i,j} \in [0, \delta]$.[4] Recall that, in this case, for all $t \geq 1$, either $W_i^t \geq W_i^{t-1}$ for $i = 1, 2$ or $W_i^t \leq W_i^{t-1}$ for $i = 1, 2$. Since $\Phi_i(\mathbf{W})$ is increasing in W_i and decreasing in W_j, for all $t \geq 1$ and for $i = 1, 2$, we have

$$\begin{aligned}
|W_i^{t+1} - W_i^t| &= |\Phi_i^\delta(\mathbf{W}^t) - \Phi_i^\delta(\mathbf{W}^{t-1})| \\
&= |\Phi_i^\delta(\mathbf{W}^t) - \Phi_i^\delta(W_i^{t-1}, W_j^t) + \Phi_i^\delta(W_i^{t-1}, W_j^t) - \Phi_i^\delta(\mathbf{W}^{t-1})| \\
&\leq \max\{|\Phi_i^\delta(\mathbf{W}^t) - \Phi_i^\delta(W_i^{t-1}, W_j^t)|, |\Phi_i^\delta(W_i^{t-1}, W_j^t) - \Phi_i^\delta(\mathbf{W}^{t-1})|\} \\
&\leq \max\{M_{i,i}, M_{i,j}\}||\mathbf{W}^t - \mathbf{W}^{t-1}|| \\
&\leq \delta||\mathbf{W}^t - \mathbf{W}^{t-1}||,
\end{aligned}$$

where the first inequality follows since Φ_i^δ is increasing in W_i and decreasing in W_j. Hence, $\{\mathbf{W}^t\}$ is a Cauchy sequence, and so it is convergent.

We now show that the finite-horizon games are convergent whenever $\delta > \underline{\delta}$. Fix $\delta > \underline{\delta}$. There are two cases to consider: (bi) $\Phi^\delta(\mathbf{0}) \in Y^\delta$, and (bii) $\Phi^\delta(\mathbf{0}) \notin Y^\delta$. Consider case (bi). By our arguments above, $\mathbf{W}(T) = (\Phi^\delta)^T(\Phi^\delta(\mathbf{0}))$ converges as $T \to \infty$.

Consider next case (bii), so that $\Phi^\delta(\mathbf{0}) \notin Y^\delta$. By equation (A2), for all $\mathbf{W}$ we have

$$\begin{aligned}
\Phi_1^\delta(\mathbf{W}) + \Phi_2^\delta(\mathbf{W}) - (W_1 + W_2) \geq \delta(W_1 + W_2) + \frac{1}{3}\underline{g}(1 - \delta(W_1 + W_2))^2 - (W_1 + W_2) \\
+ \frac{1}{3}(\underline{f} - \underline{g})(1 - \delta(W_1 + W_2))^2. \qquad \text{(O4)}
\end{aligned}$$

For all $\mathbf{W} \in X \backslash Y^\delta$, we have that

$$\delta(W_1 + W_2) + \frac{1}{3}\underline{g}(1 - \delta(W_1 + W_2))^2 > W_1 + W_2.$$

Using (O4), for all $\mathbf{W} \in X \backslash Y^\delta$ we have

$$\Phi_1^\delta(\mathbf{W}) + \Phi_2^\delta(\mathbf{W}) - (W_1 + W_2) > \frac{1}{3}(\underline{f} - \underline{g})(1 - \delta(W_1 + W_2))^2 \geq \frac{1}{3}(\underline{f} - \underline{g})(1 - \delta)^2,$$

where the last inequality follows since $W_1 + W_2 \leq 1$. This implies that, when $\Phi(\mathbf{0}) \notin Y^\delta$, there exists $t \geq 1$ such that $\Phi_1^\delta((\Phi^\delta)^t(\mathbf{0})) + \Phi_2^\delta((\Phi^\delta)^t(\mathbf{0})) \geq \underline{V}^\delta$. Hence, by our arguments above, $(\Phi^\delta)^{t+s}(\mathbf{0})$ converges as $s \to \infty$, and so the games are convergent.

Consider next part (ii). Note that when F is symmetric, both players have the same equilibrium payoffs for all deadlines, i.e. $W_1(T) = W_2(T)$ for all $T \geq 0$. Let $\hat{W}(T) = W_1(T) + W_2(T)$, and note that $\hat{W}(T) = \Psi^{T+1}(0)$ (where Ψ is the operator defined in equation (4)).

For any $\hat{W} \in [0, 1]$, define

$$H(\hat{W}) := \text{prob}(\mathbf{x} \in A(\hat{W}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W})],$$

[4]For all $\delta > \underline{\delta}$ and all $\mathbf{W} \in Y^\delta$, $\delta - \overline{f}\frac{2}{3}\delta(1 - \delta(W_1 + W_2)) \geq 0$ (see Step 2 in the proof of Proposition 2). Since $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_i} \in [\delta - \frac{\overline{f}}{3}\delta(1 - \delta(W_1 + W_2)), \delta]$ and $\frac{\partial \Phi_i^\delta(\mathbf{W})}{\partial W_j} \in [-\frac{\overline{f}}{3}\delta(1 - \delta(W_1 + W_2)), 0]$, we have that $M_{i,i} \in [0, \delta]$ and $M_{i,j} \in [0, \delta]$ for all $\delta > \underline{\delta}$.

so that $\Psi(\hat{W}) = \delta\hat{W} + H(\hat{W})(1-\delta\hat{W})$. Note that $H'(\hat{W}) \leq 0$. Indeed, $\hat{W}'' > \hat{W}'$ implies that $A(\hat{W}'') \subset A(\hat{W}')$, so for any $\hat{W}'' > \hat{W}'$,

$$\text{prob}(\mathbf{x} \in A(\hat{W}''))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W}'')] \leq \text{prob}(\mathbf{x} \in A(\hat{W}'))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W}')].$$

It then follows that $\Psi'(\hat{W}) = \delta(1 - H(\hat{W})) + H'(\hat{W})(1-\delta\hat{W}) \leq \delta < 1$ for all $\hat{W} \in [0,1]$. When $\Psi'(\hat{W}) > -1$ for all $\hat{W} \in [0,1]$, $|\Psi'(\hat{W})| < 1$ for all $\hat{W} \in [0,1]$. This implies that Ψ is a contraction, and the sequence $\{\hat{W}(T)\}$ converges to its unique fixed point. Hence, the games are convergent. ■

Proof of Proposition 6. First we prove that if F is symmetric, then the fixed point of Ψ is unique. Operator Ψ is continuous and maps $[0,1]$ onto itself, so by Brouwer's fixed point theorem, it has a fixed point.

Let $\hat{W}$ be a fixed point of Ψ. Then, $\hat{W}$ satisfies

$$\hat{W} = \frac{\text{prob}(\mathbf{x} \in A(\hat{W}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W})]}{1 - \delta + \delta\text{prob}(\mathbf{x} \in A(\hat{W}))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W})]}. \tag{O5}$$

Note that $A(\hat{W}'') \subset A(\hat{W}')$ for any $\hat{W}'' > \hat{W}'$. Therefore, for any $\hat{W}'' > \hat{W}'$,

$$\text{prob}(\mathbf{x} \in A(\hat{W}''))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W}'')] \leq \text{prob}(\mathbf{x} \in A(\hat{W}'))\mathbb{E}[x_1 + x_2 | \mathbf{x} \in A(\hat{W}')].$$

Thus, the right side of (O5) is decreasing in $\hat{W}$, and so Ψ has a unique fixed point.

Next, the sum of the players' equilibrium payoffs in a game with deadline T is $\hat{W}(T) = \Psi^{T+1}(0)$. By standard results in dynamical systems (e.g., Theorem 4.2 in **?**), under conditions (i) and (ii) in the statement of the proposition the sequence $\{\hat{W}(T)\}$ does not converge. So the games must be cycling. ■

OC Proofs and Details for Section IV

OC.1 Proofs for Stated Results

Proof of Proposition 7. We start with part (i).

Fix $\lambda \in [0,1]$, and let W_λ be the largest fixed point of $\Pi^\lambda(\cdot)$. We start by showing that there exists an SPE σ^λ in which the λ-weighted sum of players' payoffs is W_λ. Strategy profile σ^λ is as follows. Along the path of play, at each period t with status-quo $\mathbf{z}_t$, player $i = 1,2$ accepts policy draw $\mathbf{x} \in X$ if and only if

$$\lambda x_1 + (1-\lambda)x_2 + \delta(1 - x_1 - x_2)W_\lambda \geq \lambda z_1 + (1-\lambda)z_2 + \delta(1 - z_1 - z_2)W_\lambda$$

which is equivalent to

$$\lambda(x_1 - z_1) + (1-\lambda)(x_2 - z_2) \geq \delta(x_1 + x_2 - z_1 - z_2)W_\lambda.$$

If at any period t a player rejects a policy that was supposed to be accepted, then from time $t+1$ onwards both players reject all policies. Note that the payoff player i obtains from rejecting a policy at time t that should have been accepted is z_i^t. Since her continuation payoff at time t from playing according σ^λ is weakly larger than z_i^t, this strategy profile constitutes an SPE. Moreover, players' λ-weighted sum of payoffs under σ^λ is W_λ. Hence, $U_\lambda \geq W_\lambda$.

Next, we show that $U_\lambda \leq W_\lambda$. Fix $\sigma \in \Sigma$, and let $A^\sigma(h_0)$ denote the set of draws that both players accept under σ at history h_0. For each $\mathbf{x} \in X$, let $h_0^{\mathbf{x}}$ denote the history that follows h_0 if $\mathbf{x}$ is drawn and both players accept it. Then,

$$\begin{aligned}
&\lambda V_1^\sigma(h_0) + (1-\lambda)V_2^\sigma(h_0) \\
=&\text{prob}(\mathbf{x} \in A^\sigma(h_0))\mathbb{E}^\sigma[(1-\delta)(\lambda x_1 + (1-\lambda)x_2) + \delta(\lambda V_1^\sigma(h_0^{\mathbf{x}}) + (1-\lambda)V_2^\sigma(h_0^{\mathbf{x}})|\mathbf{x} \in A^\sigma(h_0)] \\
&\quad + \delta\text{prob}(\mathbf{x} \notin A^\sigma(h_0))\mathbb{E}^\sigma[\lambda V_1^\sigma(h_1) + (1-\lambda)V_2^\sigma(h_1)|\mathbf{x} \notin A^\sigma(h_0)] \qquad \text{(O6)}
\end{aligned}$$

By Lemma 1, for any $\mathbf{x} \in X$ it must be that $\lambda V_1^\sigma(h_0^{\mathbf{x}}) + (1-\lambda)V_2^\sigma(h_0^{\mathbf{x}}) \leq (\lambda x_1 + (1-\lambda)x_2) + (1-x_1-x_2)U_\lambda$. Therefore, by (O6),

$$\begin{aligned}
&\lambda V_1^\sigma(h_0) + (1-\lambda)V_2^\sigma(h_0) \\
\leq&\text{prob}(\mathbf{x} \in A^\sigma(h_0))\mathbb{E}^\sigma[\lambda x_1 + (1-\lambda)x_2 - (x_1+x_2)\delta U_\lambda|\mathbf{x} \in A^\sigma(h_0)] + \delta U_\lambda \\
\leq&\text{prob}(\mathbf{x} \in A_\lambda(U_\lambda))\mathbb{E}^\sigma[\lambda x_1 + (1-\lambda)x_2 - (x_1+x_2)\delta U_\lambda|\mathbf{x} \in A_\lambda(U_\lambda)] + \delta U_\lambda = \Pi^\lambda(U_\lambda), \quad \text{(O7)}
\end{aligned}$$

where the second inequality follows since $A_\lambda(U_\lambda) = \{\mathbf{x} : \lambda x_1 + (1-\lambda)x_2 \geq (x_1+x_2)\delta U_\lambda\}$. Since (O7) holds for any SPE σ, it must be that $U_\lambda \leq \Pi^\lambda(U_\lambda)$.

Finally, we show that $\Pi^\lambda(U) < U$ for all $U > W_\lambda$. To see why, note that

$$\Pi^\lambda(1) = \text{prob}(\mathbf{x} \in A_\lambda(1))\mathbb{E}^\sigma[\lambda x_1 + (1-\lambda)x_2 - (x_1+x_2)\delta|\mathbf{x} \in A_\lambda(1)] + \delta < 1,$$

where the strict inequality follows since, for all $\mathbf{x} \in X$,

$$\lambda x_1 + (1-\lambda)x_2 - (x_1+x_2)\delta = (x_1+x_2)(1-\delta) - (1-\lambda)x_1 - \lambda x_2 < (1-\delta).$$

Towards a contradiction, suppose that there exists $U > W_\lambda$ with $\Pi^\lambda(U) \geq U$. Since W_λ is the largest fixed point of Π^λ, it must be that $\Pi^\lambda(U) > U$. Since $\Pi^\lambda(1) < 1$, and since Π^λ is continuous, there exists $U' \in (U, 1)$ such that $\Pi^\lambda(U') = U'$, a contradiction. Hence, $\Pi^\lambda(U) < U$ for all $U > W_\lambda$. Since $U_\lambda \leq \Pi^\lambda(U_\lambda)$, it follows that $U_\lambda \leq W_\lambda$.

Now for part (ii). For $\delta < 1$ and $\lambda \in [0,1]$, let U_λ^δ denote the largest fixed point of Π^λ under discount factor δ. To prove the result, we show that for $\lambda \in \{0,1\}$, $\lim_{\delta\to1} U_\lambda^\delta = 1$. Note that this implies that payoffs $(1,0)$ and $(0,1)$ both belong in $\lim_{\delta\to1}\mathcal{V}^\delta$. Since $\mathbf{0} \in \mathcal{V}^\delta$ for all $\delta < 1$ (because the game has an SPE in which both players reject all offers), we have that $\lim_{\delta\to1}\mathcal{V}^\delta = X$.

Fix $\lambda = 1$ (the proof for $\lambda = 0$ is symmetric and omitted). For each $\delta < 1$, U_1^δ solves:

$$U_1^\delta = \frac{\text{prob}(\mathbf{x} \in A_1^\delta(U_1^\delta))\mathbb{E}[x_1|\mathbf{x} \in A_1^\delta(U_1^\delta)]}{1 - \delta + \text{prob}(\mathbf{x} \in A_1^\delta(U_1^\delta))\mathbb{E}[x_1+x_2|\mathbf{x} \in A_1^\delta(U_1^\delta)]} \qquad \text{(O8)}$$

Fix a sequence $\delta_n \to 1$, and suppose by contradiction that $\lim_{n\to\infty} U_1^{\delta_n} = k < 1$ (if needed, take a convergent subsequence). Note then that $A_1^{\delta_n}(U_1^{\delta_n}) \to A_1^* := \{\mathbf{x} \in X : x_1 \geq (x_1+x_2)k\}$. Since $k < 1$, and since f has full support, set A^* has positive measure. Moreover, since f has full support, $\mathbb{E}[x_1 + x_2|\mathbf{x} \in A^*] < \frac{1}{k}\mathbb{E}[x_1|\mathbf{x} \in A^*]$.[5] Using this in (O8), we get

$$
\begin{aligned}
k = \lim_{n\to\infty} U_1^{\delta_n} &= \lim_{n\to\infty} \frac{\text{prob}(\mathbf{x} \in A_1^\delta(U_1^\delta))\mathbb{E}[x_1|\mathbf{x} \in A_1^\delta(U_1^\delta)]}{1 - \delta + \text{prob}(\mathbf{x} \in A_1^\delta(U_1^\delta))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A_1^\delta(U_1^\delta)]} \\
&= \frac{\mathbb{E}[x_1|\mathbf{x} \in A_1^*]}{\mathbb{E}[x_1 + x_2|\mathbf{x} \in A^*]} > \frac{\mathbb{E}[x_1|\mathbf{x} \in A_1^*]}{\frac{1}{k}\mathbb{E}[x_1|\mathbf{x} \in A^*]} = k,
\end{aligned}
$$

a contradiction. Hence, $\lim_{\delta\to 1} U_1^\delta = 1$. ■

Proof of Proposition 8. Note that in this case RME payoffs are a fixed point of operator $\Phi : X \to X$, with Φ_i now given by

$$
\Phi_i(\mathbf{W}) = \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_i - (x_1 + x_2)\delta_i W_i|\mathbf{x} \in A(\mathbf{W})] + \delta_i W_i,
$$

where

$$
A(\mathbf{W}) = \left\{\mathbf{x} \in X : \text{for } i = 1, 2,\ x_i \geq \frac{\delta_i W_i}{1 - \delta_i W_i} x_{-i}\right\}.
$$

Proposition 1(ii) extends to this environment. When Assumption 3 holds, there exists $\hat{\delta} < 1$ such that, if $\delta_1 > \delta_2 > \hat{\delta}$, the game has unique RME payoffs. Moreover, as we showed in the proof of Proposition 5, when players' discount factors are sufficiently high, RME payoffs are given by $\lim_{T\to\infty} \Phi^T(\mathbf{0})$.[6]

Fix $\delta_1 > \delta_2 > \hat{\delta}$, and let $\mathbf{W}^\sigma = (W_1^\sigma, W_2^\sigma)$ denote the players' unique RME payoffs. We first show that $W_1^\sigma > W_2^\sigma$. Define the sequence $\{\mathbf{W}^T\}$ with $\mathbf{W}^T = \Phi^T(\mathbf{0})$ for each $T = 1, 2, ...$, and note that $\lim_{T\to\infty} \mathbf{W}^T = \mathbf{W}^\sigma$. Note that, for $i = 1, 2$, $W_i^1 = \Phi_i(\mathbf{0}) = \mathbb{E}[x_i]$. Since distribution F is symmetric, $W_1^1 = W_2^1$.

Next, suppose that $W_1^T \geq W_2^T$. We now show that this implies that $W_1^{T+1} > W_2^{T+1}$. Indeed, note that

$$
\begin{aligned}
W_1^{T+1} - W_2^{T+1} &= \Phi_1(\mathbf{W}^T) - \Phi_2(\mathbf{W}^T) \\
&= \text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[x_1 - x_2|\mathbf{x} \in A(\mathbf{W}^T)] \\
&\quad + (\delta_1 W_1^T - \delta_2 W_2^T)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^T)]).
\end{aligned}
$$

Since F is symmetric and since $W_1^T \geq W_2^T$, we have $\text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[(x_1 - x_2)|\mathbf{x} \in A(\mathbf{W}^T)] \geq 0$. Moreover, using $\text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[(x_1 + x_2)|\mathbf{x} \in A(\mathbf{W}^T)] < 1$, $W_1^T \geq W_2^T$ and $\delta_1 > \delta_2$, we have

$$
(\delta_1 W_1^T - \delta_2 W_2^T)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^T))\mathbb{E}[(x_1 + x_2)|\mathbf{x} \in A(\mathbf{W}^T)]) > 0.
$$

[5] Indeed, for any $\mathbf{x} \in A^*$, $x_1 \geq (x_1 + x_2)k$, and so $x_1 + x_2 \leq \frac{1}{k}x_1$. Since f has full support, $\mathbb{E}[x_1 + x_2|\mathbf{x} \in A^*] < \frac{1}{k}\mathbb{E}[x_1|\mathbf{x} \in A^*]$.

[6] While the proof of Proposition 5 is written for the case of equal discounting, the arguments can be readily extended to the case of unequal discounting.

Hence, $W_1^{T+1} > W_2^{T+1}$. Together with $W_1^1 = W_2^1$, this implies that $W_1^\sigma > W_2^\sigma$.

Next, since $\mathbf{W}^\sigma$ is a fixed point of Φ, we have

$$\begin{aligned}
W_1^\sigma - W_2^\sigma &= \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 - x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)] \\
&\qquad + (\delta_1 W_1^\sigma - \delta_2 W_2^\sigma)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)]) \\
&= \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 - x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)] \\
&\qquad + \delta_1(W_1^\sigma - W_2^\sigma)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)]) \\
&\qquad + (\delta_1 - \delta_2)W_2^\sigma(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)]) \\
&\geq \delta_1(W_1^\sigma - W_2^\sigma)(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)]) \\
&\qquad + (\delta_1 - \delta_2)W_2^\sigma(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)]),
\end{aligned}$$

where the last inequality uses $\mathbb{E}[(x_1 - x_2)|\mathbf{x} \in A(\mathbf{W}^\sigma)] > 0$, which holds since F is symmetric and since $W_1^\sigma > W_2^\sigma$. By the inequality above,

$$W_1^\sigma - W_2^\sigma \geq \frac{(\delta_1 - \delta_2)W_2^\sigma(1 - \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)])}{1 - \delta_1 + \delta_1 \text{prob}(\mathbf{x} \in A(\mathbf{W}^\sigma))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W}^\sigma)]}.$$

Using $H(\mathbf{W}) = \text{prob}(\mathbf{x} \in A(\mathbf{W}))\mathbb{E}[x_1 + x_2|\mathbf{x} \in A(\mathbf{W})]$, this is equivalent to the inequality stated in the proposition. ■

OC.2 Details for Strategic Search

In this appendix we flesh out the extension described in Section IV.C. We make the following assumptions on the sets of distributions $\mathcal{F}_\mathbf{x}$. First, for all $\mathbf{x}, \mathbf{y} \in X$, $\text{card}(\mathcal{F}_\mathbf{x}) = \text{card}(\mathcal{F}_\mathbf{y})$; i.e., all the sets $\mathcal{F}_\mathbf{x}$ have the same cardinality. Second, for all $\mathbf{x} \in X$ and all $F_\mathbf{x} \in \mathcal{F}_\mathbf{x}$ with density $f_\mathbf{x}$, there exists $F \in \mathcal{F} = \mathcal{F}_{(0,0)}$ with density f such that $f_\mathbf{x}(\mathbf{y}) = \frac{1}{(1-z_1-z_2)^2} f(P_\mathbf{x}(\mathbf{y}))$ for all $\mathbf{y} \in X(\mathbf{x})$. We further assume that there exists $\overline{f} > \underline{f} > 0$ such that, for all $f \in \mathcal{F}$, $f(\mathbf{x}) \in [\underline{f}, \overline{f}]$ for all $\mathbf{x} \in X$. Note that these assumptions are a generalization of Assumptions 1 and 2 to the new environment.

Fix an RME σ. For each $\mathbf{z} \in X$, let $V_i^\sigma(\mathbf{z})$ be player i's continuation payoff under σ when the status quo is $\mathbf{z}$ and let W_i^σ be player i's payoff at the start of the game under σ. The following result extends Lemma 1 to this environment. The proof is identical to the proof of Lemma 1, and hence omitted.

Lemma OC.1. *Fix an RME σ. For all $\mathbf{z} = (z_1, z_2) \in X$,*

$$V_i^\sigma(\mathbf{z}) = z_i + (1 - z_1 - z_2)W_i^\sigma. \tag{O9}$$

Lemma OC.1 can be used to obtain a recursive characterization of RME payoffs. Fix an RME σ. As in our baseline model, under σ player i approves a policy $\mathbf{x} = (x_1, x_2) \in X(\mathbf{z})$ when the status quo is $\mathbf{z}$ only if

$$(1 - \delta)x_i + \delta V_i^\sigma(\mathbf{x}) \geq (1 - \delta)z_i + \delta V_i^\sigma(\mathbf{z})$$

which, using Lemma OC.1, becomes

$$x_i + (1 - x_1 - x_2)\delta W_i^\sigma \geq z_i - (1 - x_1 - x_2)\delta W_i^\sigma,$$

Thus, player i accepts policy $\mathbf{x}$ when the status quo is $\mathbf{z}$ only if

$$\mathbf{x} \in A_{i,\mathbf{z}}(W_i^\sigma) = \{\mathbf{x} \in X(\mathbf{z}) : x_i \geq \ell_{i,\mathbf{z}}(x_{-i}|W_i^\sigma)\},$$

where $\ell_{i,\mathbf{z}}(x_{-i}|W_i^\sigma)$ is defined as in the main text. For any pair of payoffs $\mathbf{W} = (W_1, W_2)$ and for any $\mathbf{z} \in X$, the set $A_{\mathbf{z}}(\mathbf{W})$ defined in the main text is the set of policies that are accepted by both players when the status quo is $\mathbf{z}$, and $A(\mathbf{W})$ is the acceptance set at the start of the game.

Now suppose player $i = 1, 2$ is recognized to choose the distribution from which the policy will be drawn at the initial period. If player i chooses distribution $F \in \mathcal{F}$, she obtains payoffs equal to

$$\text{prob}_F(x \in A(\mathbf{W}))\mathbb{E}_F[x_i - (x_1 + x_2)\delta W_i|\mathbf{x} \in A(\mathbf{W})] + \delta W_i.$$

For any $\mathbf{W} \in X$ and for $i = 1, 2$, let

$$F^*_{\mathbf{W},i} \in \arg\max_{F \in \mathcal{F}} \text{prob}_F(x \in A(\mathbf{W}))\mathbb{E}_F[x_i - (x_1 + x_2)W_i|\mathbf{x} \in A(\mathbf{W})],$$

and let $F^*_{\mathbf{W}} := \frac{1}{2}F^*_{\mathbf{W},1} + \frac{1}{2}F^*_{\mathbf{W},2}$. Note that the initial period policy is drawn from distribution $F^*_{\mathbf{W}}$.

Define the operator $\Phi^S : X \to X$ as follows: for $i = 1, 2$ and for all $\mathbf{W} \in X$,

$$\Phi_i^S(\mathbf{W}) = \text{prob}_{F^*_{\mathbf{W}}}(x \in A(\mathbf{W}))\mathbb{E}_{F^*_{\mathbf{W}}}[x_i - (x_1 + x_2)\delta W_i|\mathbf{x} \in A(\mathbf{W})] + \delta W_i.$$

Let $\mathbf{W}^*$ denote the players' RME payoffs at the start of the game. The following result extends Proposition 1 to the current environment – the proof uses the same arguments as the proof of Proposition 1, and hence we omit it.

Proposition OC.1. *An RME exists, and the players' equilibrium payoffs under an RME are a fixed point of* Φ^S.

This characterization of equilibrium payoffs can be used to generalize the main results in the main text to the current environment. First, any RME features inefficient delays. Second, the acceptance regions are nested, and the distribution over long-run outcomes that an RME induces at a subgame starting with status quo payoff $\mathbf{z}$ has support equal to $\{\mathbf{x} \in X : x_1 + x_2 = 1\} \cap A_{\mathbf{z}}(\mathbf{W})$. Therefore, RME also display path-dependence. It can also be shown that Proposition 4 continues to hold in this setting, so the RME outcome also becomes deterministic in the limit as $\delta \to 1$.[7]

[7] The proofs of all of these results are available upon request.