

License Complementarity and Package Bidding: U.S. Spectrum Auctions

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Appendix: For Online Publication

1) More Bidding Patterns

Figure A1 reports the distribution of the first round when license winners in Block B placed their bids on the licenses won. Around 47% of the winners started bidding on the licenses won in the first round and more than 75% of them started bidding on the licenses won in the first 20 rounds. Only a few bidders started bidding late on a license and won the licenses eventually. We take this as further evidence of straightforward bidding in this auction.

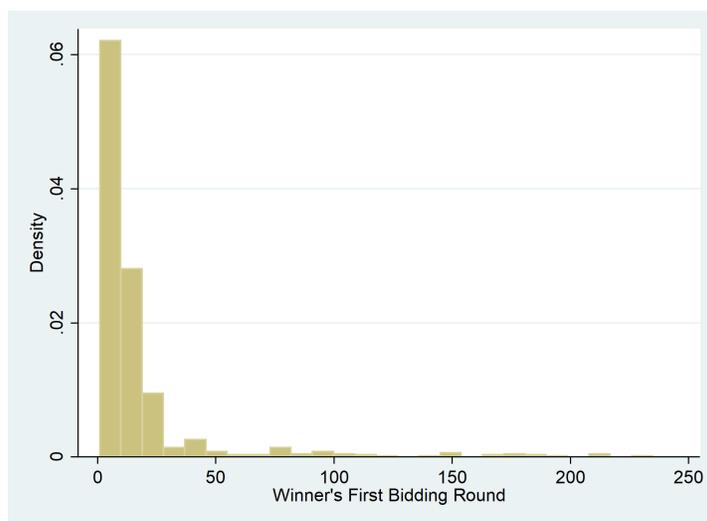


Figure A1. : Distribution of Winner's First Bidding Round

Table A1 reports the number of licenses that a bidder bid on across blocks in a given round of Auction 73. We take a sub-sample that starts from round 21 and contains only bidders who placed at least one bid in a round in the license's associated block. There are 5,358 such bidder-block-round observations. As shown in the table, bidders often placed multiple bids in the same round. A bidder bid on a maximum of 197 licenses in the same round. This is clear evidence of bidders' expected complementarity over multiple licenses.

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Table A1—: A Bidder Places Multiple Bids in a Round

# Licenses	Frequency	Percentage
1	3,219	59.78
2	922	17.12
3 to 5	745	13.83
6 to 10	275	5.11
11 to 20	119	2.21
21 to 197	105	1.95
Total	5,385	100

2) Summary Statistics on Cross-Blocks Bidding Behavior

We analyze Block B bidders' complementarity indices within the block and across blocks in different bidding status. A bidder had three possible bidding status for any license in a given round: bidding, being the provisional winner, or not bidding. For each bidder at each license-round observation, we calculate the within-block and cross-block complementarity indices (defined by Equation 10). The within-block complementarity-index measures the complementarity between a Block B license and all other block B licenses. The cross-block complementarity index measures the complementarity between a Block B license and all licenses in Block A/C/D/E. In Table A2, column (1) reports Block B bidders' average within-block index across each bidding status and bidder type combination; column (2) reports Block B bidder's average cross-block index across each bidding status and bidder type combination; column (3) reports the ratio of the first two columns; and column (4) reports the number of bidder-license-round used in the calculation.

Table A2—: Within- and Cross- Blocks Complementarity in Block B

Status	Bidder	(1) $\tau(WithinBlock)$	(2) $\tau(CrossBlock)$	(3) $\frac{\tau(WithinBlock)}{\tau(CrossBlock)}$	(4) # of obs
Bidding	Large	0.0115	0.0075	1.5	8,752
	Medium	0.0017	0.0006	2.6	7,370
	Small	0.0016	0.0003	5.1	9,037
Prov. Winner	Large	0.0420	0.0055	7.6	79,992
	Medium	0.0030	0.0007	4.5	34,342
	Small	0.0022	0.0008	2.9	73,150
Not Bidding	Large	0.0065	0.0132	0.5	160,856
	Medium	0.0004	0.0012	0.3	229,988
	Small	0.0009	0.0009	1.0	282,333

As shown in Table A2, when an average bidder bid in a round on a license, its within-block complementarity index was much higher than the cross-block complementarity index (the column 3 ratios are far greater than 1). Similarly, when a bidder became a provisional winner of a license, its within-block complementarity index was much higher than its cross-block complementarity index. In contrast, when a bidder did not bid on a license, its within-block complementarity index was much lower than its cross-block complementarity. The bidding patterns in Table A2 indicate that the within-block complementarity drove bidders' bidding decisions in this auction; therefore, we

focus on within-block complementarity in this study.

3) Comparison of Different Complementarity Indices

In this paper, we do not incorporate the measure of travel complementarity, as in Fox and Bajari (2013) and Yeo (2009), for the following reasons: first, their measures of license complementarity are at the BTA or CMA level, but our measure is at the county level. If we construct measures of travel complementarity at the county level, there may be substantial bias in the measure of complementarity because there will be high complementarity between counties with an airport, but no complementarity in the counties with no airport even though they are close to an airport. Second, we deem travel complementarity not as important as distance complementarity. This argument is consistent with the empirical findings in Fox and Bajari (2013)'s empirical results, where the two travel complementarity parameters are not significant. Third, Fox and Bajari (2013) have shown that the three complementarity measures (one distance complementarity index and two travel complementarity indices) are highly correlated.

Table A3—: Summary of Complementarity by Market Type

Market Type		(1)		(2)		(3)	
		County-level		CMA-level		CMA-level 2	
		mean	s.d	mean	s.d	mean	s.d
CMA	Within	0.415	2.526	-	-	-	-
	Between	0.003	0.047	0.004	0.067	0.004	0.130
BEA	Within	3.018	7.904	1.999	4.683	4.913	9.140
	Between	0.032	0.232	0.054	0.376	0.033	0.492
REA	Within	136.054	17.143	123.562	39.999	156.386	40.416
	Between	12.245	10.220	16.102	11.705	2.792	3.943
National license	Benchmark	1,000		1,000		1,000	

Note: Complementarity listed in the table is complementarity indices $\times 1000$.

Table A3 compares three measures of market complementarity. All of them are based on the geographic complementarity function in Fox and Bajari (2013) (with $\delta = 2$). Measure 1 is measured at the county level. This is the one we discussed in the main text and used as our empirical input. Measures 2 and 3 are at the CMA level. The difference between measures 2 and 3 is that measure 3 follows Yeo (2009) and treats the distance between CMA l and l' as the minimum distance between two counties within CMA l and CMA l' (These two counties must belong to the same state). These summary statistics show that all three measures are very close to each other.

4) Constructing Cell Phone Tower Variable

The FCC maintains a cell phone tower registration database that records all tower ownership or usage information.¹ We use two data sets from this database: the EN data set and the RA data set. The EN data set records the owner of a tower, whereas the RA data set matches each tower to a county. We count the number of towers a firm has in a county after we merge these two data

¹The website is <http://wireless.fcc.gov/uls/>.

sets. Lastly, we manually match firm names in the tower registration database with the identity of the bidders in the auction data to obtain the number of towers a bidder has in a county.

A drawback of the data is that ownership changes over time, but the FCC does not keep historical records database. So, what we use in this paper is the tower ownership structure in late 2016, when we started the project. As Auction 73 happened in 2008, there may be changes in the number of towers that a bidder has in a county during the eight-year gap. These changes include: change of ownership, new towers, merger between bidders, etc. Still, these data are the best data we could get to measure bidders' stock of cell phone towers in different counties. We think that this variable is a combination of towers that had been built before Auction 73 and towers that would be built after Auction 73, which is a good measure of bidder heterogeneity.

5) Summary Statistics of Variables Used for Estimation

Table A4 reports summary statistics of variables used for estimation.

Table A4—: Summary Statistics for Estimation

Panel I: Table 5, Belief Estimation, # obs: 813,616						
Variable	Mean	Median	Std	Min	Max	
Complementarity	0.006	0.000	0.038	0	1.41	
Complementarity Sq	0.002	0.000	0.037	0	1.99	
# <i>Competitor</i> = 1	0.775	1.000	0.418	0	1	
# <i>Competitor</i> = 2	0.004	0.000	0.059	0	1	
# <i>Competitor</i> ≥ 3	0.001	0.000	0.025	0	1	
Pop*Bandwidth	0.023	0.008	0.062	0.00	0.65	
$\ln(\frac{Pop_c}{Area_c} + 1)$	1.061	0.824	0.835	0.01	4.72	
$\ln(\frac{Tower_{ic}}{Area_c} + 1)$	1.433	0.000	3.115	0	10.89	
ln Upfront Pay	17.130	17.859	2.875	7.90	20.60	
Round	141.000	141.000	69.570	21	261	
# Win Round	21.450	0.000	52.209	0	256	
Panel II: Table 6, Complementarity Effect Estimation, # obs: 198,997						
	Mean	Median	Std	Min	Max	
Price (no bid)	1,555,728	315,750	10,600,000	4,575	937,000,000	
Complementarity Index (no bid)	0.001	0.000	0.003	0.000	0.202	
Price (bid)	1,151,258	277,000	8,880,105	4,875	804,000,000	
Complementarity Index (bid)	0.001	0.000	0.003	0.000	0.155	
Panel III: Table 7, Stand-alone Value Estimation, # obs: 170,246						
	Mean	Median	Std	Min	Max	
Bid this round	0.041	0.000	0.198	0.000	1.000	
$\ln(Pop_l)$	-7.584	-7.506	0.996	-12.061	-2.914	
$\ln(\frac{Pop_l}{Area_l} + 1)$	0.785	0.625	0.615	0.005	3.917	
$\ln(\frac{Tower_{il}}{Area_l} + 1)$	0.904	0.000	2.583	0.000	10.892	
$\ln(UpPay_i)$	17.737	17.553	1.669	7.901	20.601	
Expected Comple. - price (in Billion \$)	-0.003	0.000	0.022	-0.973	0.139	

6) Belief Estimation Result

Table A5 reports estimates of Equation (11). This table is structured the same way as Table 5.

Table A5—: Belief Estimation Result

	Bid			Provisional Winner		
	(1) Large	(2) Medium	(3) Small	(4) Large	(5) Medium	(6) Small
Complementarity	15.91*** (5.765)	78.99 (77.91)	329.8*** (30.46)	-10.27*** (1.097)	15.23*** (4.555)	170.5*** (9.932)
Comple. Sq	-164.5*** (53.05)	-17,490 (18,348)	-16,590*** (3,184)	1.052*** (0.369)	-60.16*** (17.62)	-6,718*** (740.2)
# Competitor = 1	0.194 (0.231)	0.781 (0.603)	-0.647*** (0.249)	-1.192*** (0.099)	-1.194*** (0.110)	-1.007*** (0.070)
# Competitor = 2	-0.071 (0.253)	0.362 (0.615)	-0.940*** (0.259)	-0.555** (0.228)	-	-1.560*** (0.236)
# Competitor ≥ 3	-	0.0414 (0.706)	-0.949*** (0.300)	-1.786** (0.722)	-0.394 (0.748)	-1.298*** (0.404)
Pop*Bandwidth	3.968*** (1.449)	5.232* (2.808)	-81.360*** (7.491)	19.75*** (2.035)	7.283 (4.737)	-0.141 (3.085)
$\ln(\frac{Pop_c}{Area_c} + 1)$	-0.248*** (0.065)	-0.344*** (0.061)	-0.094** (0.038)	0.060*** (0.021)	-0.231*** (0.030)	-0.300*** (0.015)
$\ln(\frac{Tower_{ic}}{Area_c} + 1)$	0.042*** (0.010)	-0.028** (0.013)	-	0.029*** (0.003)	-0.0278*** (0.006)	-
ln(Upfront Pay)	-3.656*** (0.146)	-0.001 (0.019)	0.027*** (0.010)	-2.285*** (0.047)	-0.071*** (0.008)	0.042*** (0.003)
Round/10	0.006 (0.020)	0.046*** (0.008)	0.012*** (0.004)	0.235*** (0.016)	0.066*** (0.004)	0.022*** (0.002)
# Win Round				-0.005*** (0.002)	0.017*** (0.001)	0.012*** (0.259e-3)
Constant	73.88*** (3.008)	-1.165* (0.658)	0.305 (0.287)	46.15*** (0.968)	1.246*** (0.121)	-0.192*** (0.056)
# obs	1,706	1,779	3,664	74,600	29,859	68,215

7) Construction of Criterion Functions

Below, we discuss the construction of the criterion functions according to Equation (20). All inequalities belong to one of the following four categories:

Cat. 1: If $DS(l, t, t') > 0$ and $P_{lt} - P_{lt'} > 0$, $\beta_i \geq \frac{E[z(DS(l, t, t'))(P_{lt} - P_{lt'})]}{E[z(DS(l, t, t'))DS(l, t, t')]}$. This is a lower bound of the complementarity coefficient β_i . If bidder i does not bid on license l when the price is low (in round t') but starts to bid on the license when the price is high (in round t), the increase in the expected complementarity contribution of this license must be higher than the increase in price, which generates a lower bound for complementarity.

Cat. 2: If $DS(l, t, t') < 0$ and $P_{lt} - P_{lt'} < 0$, $\beta_i \leq \frac{E[z(DS(l, t, t'))(P_{lt} - P_{lt'})]}{E[z(DS(l, t, t'))DS(l, t, t')]}$. This is an upper bound of the complementarity effect. If bidder i bids on on license l when the price is low (in

round t) but stops bidding on this license when the price is high (in round t'), the increase in the expected complementarity contribution of this license must be lower than the increase in price, which generates an upper bound for complementarity.

Cat. 3: If $DS(l, t, t') > 0$ and $P_{lt} - P_{lt'} \leq 0$, $\beta_i \geq$ a non-positive number. However, it is uninformative because we assume that $\beta_i \geq 0$. So, inequalities in Cat 3 do not provide us with any new information.

Cat. 4: If $DS(l, t, t') < 0$ and $P_{lt} - P_{lt'} \geq 0$, $\beta_i \leq 0$. This contradicts our model because we assume that $\beta_i \geq 0$.

To estimate our model, we drop inequalities that either provide no information (Cat. 3) or contradict our model (Cat. 4) and use only the inequalities in Cat. 1 and Cat. 2 to estimate the complementarity effect. These two sets of inequalities identify the complementarity coefficient β_i .

8) Robustness to Table 6

We imposed two behavior assumptions on bidders: **BA1** and **BA2**. **BA1** is straightforward to establish: if a bidder bids on a license, then: 1) the marginal contribution of the license to the set of licenses that the bidder believes it will win is higher than the MAB on the license; and 2) the license is under the bidder's eligibility constraint. **BA2**, however, warrants extra discussion. The actions of "not bid" or "stop bidding" have more possibilities than we allow under the "straightforward" bidder assumption. There are three potential reasons that a bidder chooses not to bid on a license: 1) the marginal contribution of the license is lower than the MAB; 2) the bidding units of the license are higher than the remaining eligibility of the bidder; 3) the bidding units of the license, although lower than the bidder's remaining eligibility, are too low to satisfy the FCC's activity requirement, so the bidder may want to bid on other licenses instead to maintain its eligibility. Our baseline estimation (results shown in Table 6) incorporate both reason (1) and reason (2), but does not consider reason (3). In this robustness check, we add one more restriction: the bidding units of the license need to be higher than the additional units required to maintain the eligibility level of the bidder (but still lower than the unused eligibility of the bidder). Note that this restriction substantially reduces the number of observations used in the estimation, which is probably why we have much wider confidence intervals in Table A6.

Table A6—: Estimates of Complementarity Effect (in billion \$)

	(1) Full Sample	(2) Large	(3) Medium	(4) Small
β	1.04	1.23	0.5	1.22
95% CI	[0.73 , 1.34]	[0.55 , 1.75]	[0.42 , 0.74]	[0.67 , 1.31]
# LBs	1,897	127	827	943
# Ubs	4,169	33	1614	2,522

Note: The unit of observation is a bidder-license-round-pair. For bidder i , we include a bidding round and not-bidding round as a pair for license l .

9) Expected Complementarity and Bidding Patterns

Table A7 presents the mean and median value of a license's marginal contribution in the expected complementarity index (as defined by Equation 4) when a bidder bid on the license and when a bidder did not bid on the license.

Table A7—: A License’s Marginal Contribution to the Expected Complementarity Index

(a) When a bidder bids on a license					
	Mean	Median	s.d	min	max
Large	0.0110	0.0055	0.0173	0.0001	0.1841
Medium	0.0004	0.0002	0.0007	0.0000	0.0067
Small	0.0008	0.0002	0.0015	0.0000	0.0427
(b) When a bidder does not bid on a license					
	Mean	Median	s.d.	min	max
Large	0.0051	0.0036	0.0083	0.0000	0.3165
Medium	0.0006	0.0002	0.0012	0.0000	0.0462
Small	0.0010	0.0002	0.0020	0.0000	0.0787
(c) Difference between (a) and (b)					
	Mean	Median			
Large	0.00586	0.00198			
Medium	-0.00021	-0.00005			
Small	-0.00019	-0.00001			

Large bidders had the highest difference between the expected complementarity index when they bid and the expected complementarity index when they did not bid, followed by small bidders. The value for medium-sized bidders was close to zero. These bidding patterns indicate that large bidders were more likely to bid when the gain in expected complementarity was higher. However, medium-sized bidders’ bidding decisions were much less affected by the gain in expected complementarity. This is consistent with our estimates of β_i : large bidders displayed the highest value for complementarity, small bidders the second highest, and medium-sized bidders the lowest.

10) Counterfactual Simulation Procedure

This section describes the procedure we use to conduct one iteration of our counterfactual simulation.

Begin: outerloop

Step 1. For each bidder, take random draws of all licenses’ stand-alone values from the truncated normal distribution (Details discussed in Section 6.3).

Step 2. We start with round $t = 1$. If $t = 1$, predict the probability of each bidder ultimately winning each license if the license is in its bidding set (there is no provisional winning set). If $t > 1$, calculate the marginal contribution of each license to the complementarity index of its provisional winning set in the last round, and predict the probability of each bidder ultimately winning each license if the license is its bidding set or provisional winning set according to Equation 11).

Step 3. **Begin: Innerloop:** Compute the minimum bidding sets of each bidder. Here, we make use of the supermodularity property in this game and iterate to obtain a bidding set. In any iteration k , when we compute the minimum bidding set of a bidder, we compute the marginal contribution of all licenses not in the current bidding set towards the bidding set in iteration $k - 1$ plus its provisional winning set in round $t - 1$. We then move all licenses with positive marginal contribution to the current bidding set. We iterate until no licenses outside the bidding set make a positive contribution to the current bidding set. We compute the maximum bidding sets of each

bidder similarly (Details discussed in Section 6.4). **End: Innerloop**

Step 4. At the end of round t , each license's provisional winner is determined. When there is only one bid on a license, this bidder becomes the provisional winner of the license. When there are multiple bids on a license, we take a random draw to decide the winner, and all bidders on this license have an equal probability of becoming the provisional winner.

Step 5. Price increases by 10% for round $t + 1$ when there is a new bid on this license in round t .

Update $t = t + 1$ and iterate Steps 2-5 until no one places a new bid.

Step 6. Auction ends if no one places a new bid.

End: outerloop

11) Counterfactual Results with Iterated Bidder Belief

This section reports counterfactual results with bidders' beliefs about winning licenses iterated to convergence. In the main text, we report simulation results using bidders' estimated beliefs from observed data. In this appendix, the simulation design is the same as reported in the main text, except that we incorporate an iterating process of bidder belief. In iterations we re-estimate the bidder belief function and re-simulate auction outcomes with this updated belief until the bidder belief function converges.

Algorithm

We describe the algorithm for the belief iterating process below. In the initial iteration (iteration 0), we simulate the entire auction process with the estimated belief (from data) and obtain the simulated license allocation. We then re-estimate the bidder belief function as specified in Equation (11) with the new license allocation and obtain the updated belief function in iteration 0. In any iteration $r \geq 1$, we simulate the auction with the updated bidder belief in iteration $r - 1$. We then update the bidder belief function in iteration r using the simulated auction allocation in iteration r . We repeat this process until the updated bidder belief function is sufficiently close to the bidder belief function in the previous round.

Bidder Belief Function Specification

To facilitate the convergence process and increase convergence rate, we simplify the belief function as specified in Equation (11) and discretize state space. We include only key determinants of the belief function: the number of competitors, the expected complementarity in the last round and the market size. We further discretize these variables and include five dummy variables in the belief function, representing: (1) $\#Competitor = 1$; (2) $\#Competitor = 2$; (3) $\#Competitor \geq 3$; (4) $Complementarity \geq 0.0004046$; and (5) $Pop \geq 0.0086$.²

Simulation Results

We conduct five simulations in the paper, corresponding to five packaging policies: (a) CMA, (b) pure BEA package, (c) pure REA package, (d) mixed BEA package, and (e) mixed REA package. We conduct 500 simulations with the iterated belief process. Under policies (a), (d) and (e), it is very difficult for the belief function to converge — the convergence rate is usually below 10%. Policies (b) and (c) fare much better in terms of the convergence rate. When all bidders bid on their minimum bidding sets, the pure BEA (REA) package converges in 82.6% (92.4%) of the iterations. When all bidders bid on their maximum bidding sets, pure BEA (REA) package converges in

²The median expected complementarity value in the sample is 0.0004046 and the median population in the market is 0.0086.

72.4% (92.4%) of the iterations. We believe that the main reason for the low convergence rate under policies (a), (d) and (e) is the curse of dimensionality problem. For a pure BEA package, a bidder has 176 licenses to choose in any round of the auction; for a CMA package, the number of licenses is 734; for a mixed BEA package it is (734+ 176). It is too computationally intensive for us to improve the convergence rate. To err on the side of caution, we only report results for the pure BEA package and pure REA package.

Tables A8 to A10 summarize the simulation results. In all tables, the first two columns report results for the minimum bidding set while the last two columns report results for the maximum bidding set. Table A8 reports the magnitude of bidders' exposure problem at the end of the auction if all bidders bid on their minimum (maximum) bidding sets in all rounds of the auction. Table A9 reports the bidder surplus (in top panel) and FCC revenue (in bottom panel) at the end of the auction if all bidders bid on their minimum (maximum) bidding sets in all rounds of the auction. Table A10 reports the final license allocation at the end of the auction if all bidders bid on the minimum (maximum) bidding set in all rounds. As shown in Tables A8 to A10, simulation results with the iterated bidder belief process are very similar to the corresponding findings reported in the Tables 9 to 14. With the caveats of the convergence problem under the alternative package policies (a), (d) and (e), we consider our results for pure package policies robust to whether we iterate bidder beliefs.

Table A8—: Pure Package, Exposure Problem in the Last Round (in million \$)

		Min Bidding Set		Max Bidding Set	
		BEA	REA	BEA	REA
"Overbidding"	Large Bidders	41.66	1.93	42.07	1.93
	Medium Bidders	0.03	0.00	0.03	0.00
	Small Bidders	7.72	0.00	7.72	0.00
	Sum	49.40	1.93	49.82	1.93
"Underbidding"	Large Bidders	1,193.50	262.02	1,194.40	262.02
	Medium Bidders	369.65	0.00	369.62	0.00
	Small Bidders	463.27	0.00	463.28	0.00
	Sum	2,026.40	262.02	2,027.30	262.02
Total Exposure	Large Bidders	1,235.20	263.95	1,236.50	263.95
	Medium Bidders	369.68	0.00	369.65	0.00
	Small Bidders	470.99	0.00	470.99	0.00
	Sum	2,075.90	263.95	2,077.10	263.95

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Table A9—: Pure Package, Bidder Surplus and FCC Revenue (in billion \$)

		Min Bidding Set		Max Bidding Set	
		BEA	REA	BEA	REA
Bidder Surplus	Large	6.75	3.36	6.76	3.36
	Medium	0.00	0.00	0.00	0.00
	Small	0.02	0.00	0.02	0.00
	Sum 1	6.77	3.36	6.78	3.36
FCC Revenue	Large	13.31	18.32	13.31	18.32
	Medium	0.00	0.00	0.00	0.00
	Small	0.01	0.00	0.01	0.00
	Sum 2	13.32	18.32	13.32	18.32
Social Surplus	Sum 1 + Sum 2	20.09	21.68	20.10	21.68

Table A10—: Pure Package, License Allocation

	Min Bidding Set		Max Bidding Set	
	BEA	REA	BEA	REA
Market Share and HHI (Population Weighted)				
Market Share (Large)	98.18	100.00	98.18	100.00
Market Share (Medium)	0.12	0.00	0.12	0.00
Market Share (Small)	1.70	0.00	1.70	0.00
HHI (population)	6,096	7,156	6,096	7,156
Market Share and HHI (Unweighted)				
Market Share (Large)	95.73	100.00	95.73	100.00
Market Share (Medium)	0.61	0.00	0.61	0.00
Market Share (Small)	3.66	0.00	3.66	0.00
HHI (licenses)	6,033	7,447	6,033	7,447