Online Appendix

International Sanctions and Limits of Lerner Symmetry

Oleg Itskhoki and Dmitry Mukhin

Consider the planner's problem

$$\max_{C_1^*, C_2^*, Y_1^*, Y_2^*} U(C_1^*, C_2^*) \quad \text{s.t.} \quad P_1^* C_1^* + \frac{P_2^* C_2^*}{R^*} = Q_1^* Y_1^* + \frac{Q_2^* Y_2^*}{R^*}, \quad F(Y_1^*, Y_2^*) = 0,$$

with isoelastic preferences and CES production frontier:

$$U(C_1^*, C_2^*) = \frac{1}{1-\frac{1}{\sigma}} \left[C_1^{*1-\frac{1}{\sigma}} + \beta C_2^{*1-\frac{1}{\sigma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}}, \qquad F(Y_1^*, Y_2^*) = a_1^{-\frac{1}{\theta}} Y_1^{*\frac{\theta+1}{\theta}} + a_2^{-\frac{1}{\theta}} Y_2^{*\frac{\theta+1}{\theta}} - 1,$$

where $a_1 + a_2 = 1$ and $\sigma, \theta > 0$. The first-order conditions characterize the optimal intertemporal choice of consumption and production:

$$\frac{C_2^*}{C_1^*} = \left(\beta R^* \frac{P_1^*}{P_2^*} \right)^{\sigma}, \qquad \frac{Y_2^*}{Y_1^*} = \frac{a_2}{a_1} \left(\frac{Q_2^*}{R^* Q_1^*} \right)^{\theta}.$$

Substitute the latter condition into the production constraint to solve for Y_1^* and Y_2^*. Combining with the optimal consumption smoothing and the country's budget constraint, this leads to the welfare function

$$V = \frac{\sigma}{\sigma - 1} \left[P_1^{*1-\sigma} + \beta^{\sigma} \left(\frac{P_2^*}{R^*} \right)^{1-\sigma} \right]^{\frac{1}{\sigma-1}} \left[a_1 Q_1^{*\theta+1} + a_2 \left(\frac{Q_2^*}{R^*} \right)^{\theta+1} \right]^{\frac{1}{\theta+1}}.$$

Given the focus on the frontloaded shocks, rewrite the welfare briefly as

$$\log V = \frac{1}{\sigma - 1} \log \left[\gamma + P_1^{*1-\sigma} \right] + \frac{1}{\theta + 1} \log \left[\alpha + Q_1^{*\theta+1} \right] + \log \frac{\sigma a_1}{\sigma - 1},$$

where $\gamma \equiv \beta^{\sigma} \left(\frac{P_2^*}{R^*} \right)^{1-\sigma}$, $\alpha \equiv \frac{a_2}{a_1} \left(\frac{Q_2^*}{R^*} \right)^{\theta+1}$. The first-order derivatives of $\log V$ are given by

$$\frac{\partial \log V}{\partial \log P_1^*} = -\frac{P_1^{*1-\sigma}}{\gamma + P_1^{*1-\sigma}}, \qquad \frac{\partial \log V}{\partial \log Q_1^*} = \frac{Q_1^{*\theta+1}}{\alpha + Q_1^{*\theta+1}}$$

and the second-order derivatives are

$$\frac{\partial^2 \log V}{(\partial \log P_1^*)^2} = \frac{(\sigma - 1)\gamma P_1^{*1-\sigma}}{\left(\gamma + P_1^{*1-\sigma} \right)^2}, \qquad \frac{\partial^2 \log V}{(\partial \log Q_1^*)^2} = \frac{(\theta + 1)\alpha Q_1^{*\theta+1}}{\left(\alpha + Q_1^{*\theta+1} \right)^2}, \qquad \frac{\partial^2 \log V}{\partial \log P_1^* \partial \log Q_1^*} = 0.$$

Given the CES structure, the share of first-period revenues in total discounted income Φ and the share of first-period spendings in total discounted expenditures Ω are equal

$$\Phi = \frac{Q_1^{*\theta+1}}{\alpha + Q_1^{*\theta+1}}, \qquad \Omega = \frac{P_1^{*1-\sigma}}{\gamma + P_1^{*1-\sigma}},$$

which allows us to express the derivatives of V in terms of Φ and Ω. The second-order expansion of the welfare can then be written as

$$\mathrm{d}\log V = -\Omega\,\mathrm{d}\log P_1^* + \Phi\,\mathrm{d}\log Q_1^* + \frac{1}{2}\Omega(1-\Omega)(\sigma-1)(\mathrm{d}\log P_1^*)^2 + \frac{1}{2}\Phi(1-\Phi)(\theta+1)(\mathrm{d}\log Q_1^*)^2.$$

Given the definitions $S_1^* \equiv Q_1^*/P_1^*$ and $\tilde{R}^* \equiv R^* P_1^*/P_2^*$, the first-order terms can be decomposed into the income and substitution effect

$$\mathrm{d}\log V = \Phi\,\mathrm{d}\log S_1^* + (\Phi - \Omega)\,\mathrm{d}\log \tilde{R}^*.$$

If at the point of approximation, the country does not borrow or save in the first period, then $\Omega = \Phi$ and the expansion simplifies to

$$\mathrm{d}\log V = \Omega\,\mathrm{d}\log S_1^* + \frac{1}{2}\Omega(1-\Omega)\Big[(\sigma-1)(\mathrm{d}\log P_1^*)^2 + (\theta+1)(\mathrm{d}\log Q_1^*)^2\Big].$$