Optimal Lending Contracts with Retrospective and Prospective Bias

By J. Aislinn Bohren and Daniel N. Hauser

Online Appendix

Proof of Corollary 1.

A correctly specified entrepreneur uses h_B and ρ_B. Note that $V_{h_B} = V_{\rho_B}$ and $m_{h_B} = 1/2$. From Eq. (7), this implies $r^*(h_B, \rho_B) = 0$. When the entrepreneur uses h and ρ_h, she correctly anticipates her posterior beliefs. Therefore, $V_{\rho_h} = V_h$ and $m_{\rho_h} = m_h$. Given that the forecast is plausible, $m_{\rho_h} = 1/2$. Together this implies $m_h = 1/2$. Again it follows that $r^*(h, \rho_h) = 0$. The result for c^* follows from Eq. (8), while the result on the lender's expected profit follows from substituting $r^*(h, \rho_h) = 0$ into the profit expression in the proof of Proposition 1. □

Proof of Corollary 2.

From Proposition 1, given $\hat{\rho}_\theta$, $r^*(h_B, \hat{\rho}_\theta) = \frac{\theta-1}{7\theta+5}$ and $c^*(h_B, \hat{\rho}_\theta) = \frac{(\theta+1)(7\theta+5)}{8(2\theta+1)^2}$. This follows from $V_{h_B} = 1/12$ when $d\rho_B = 1$ and $V_{\hat{\rho}} = 1/(8\theta + 4)$. When $\theta = 1$, $r^*(h_B, \rho_B) = 0$ and $c^*(h_B, \rho_B) = 1/3$. From these expressions, it immediately follows that $r^*(h_B, \hat{\rho}_\theta) < 0$ for $\theta < 1$ and $r^*(h_B, \hat{\rho}_\theta) > 0$ for $\theta > 1$. Further, $c^*(h_B, \hat{\rho}_\theta)$ is decreasing in θ. □