Rethinking Detroit

Online Appendix B

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Contents

1 The Model

The city consists of different locations that are composed of potentially two types of areas: business areas and residential areas. In some instances, locations may be entirely residential or commercial. Residential areas provide housing services and residential amenities, while business areas are used for production. There are J locations, indexed by $j \in \{1, ..., J\}$ (or i). We denote the amount of land used in different areas by T_j^b or T_j^r depending on whether the land is used for business or residential purposes. Furthermore, we denote the amount of land zoned for residential development by $\overline{T}_j^r$, and likewise the amount of land zoned for business use by $\overline{T}_j^b$, so that $\overline{T}_j^r$ and $\overline{T}_j^b$ represent upper bounds on residential and business development, respectively. Actual developed residential land, T_j^r, is determined in equilibrium, but we assume that business land is always developed, so that in all locations $T_j^b = \overline{T}_j^b$.

In equilibrium, residential areas may be populated or vacant. Some of the residential vacant areas may also result from simply not being zoned for residential use. However, allowing such areas to be used for residential purposes does not necessarily imply that they will be populated in equilibrium; they may remain vacant. Similarly, not all J locations will necessarily contain commercial activity if some locations are primarily residential - there is one census tract in our data with no measured employment. We denote by $\underline{J} \leq J$ the number of locations with some residential population, which leaves $J - \underline{J}$ vacant residential locations, and by $I \leq J$ the number of locations with active business or commercial areas.

1.1 Firms

All firms produce a single consumption good and choose where and how much to produce of that good. Production per unit of land in the business district of location j is given by

$$\frac{Y_j}{T_j^b} \equiv y_j = a\left(\frac{L_j}{T_j^b}; j\right)\left(\frac{L_j}{T_j^b}\right)^{\beta} \equiv \left(A_j l_j^{\alpha}\right) l_j^{\beta}. \tag{1}$$

Let $A\left(l_j\right)$ denote a production externality in location j which is assumed local (so only employment in j affects the productivity of businesses in j). For simplicity, specify $A\left(l_j\right) = A_j l_j^{\alpha}$ with $\alpha > 0$ and let w_j be the wage in location j. Then the problem of firm k (assuming that firms are small and do not internalize the externality) is

$$\max_{l_{kj}} a\left(l_j; j\right) l_{kj}^{\beta} - w_j l_{kj}, \tag{2}$$

where l_{kj} denotes firm k's choice of workers per unit of land, so that

$$\beta a\left(l_j; j\right) l_{kj}^{\beta-1} = w_j. \tag{3}$$

Since all producers at j are identical, in equilibrium $l_{kj} = l_j$ for all k. Hence, using $a\left(l_j; j\right) = A_j l_j^{\alpha}$,

$$l_j = \left(\frac{A_j \beta}{w_j}\right)^{\frac{1}{1-\beta-\alpha}}. \tag{4}$$

Since firms compete for land and are willing to bid for business land at j until they make zero profits, the commercial bid rent faced by firms in j then is

$$q_j^b = (1-\beta) A_j^{\frac{1}{1-\beta-\alpha}} \left(\frac{\beta}{w_j}\right)^{\frac{\beta+\alpha}{1-\beta-\alpha}}, \tag{5}$$

and labor demand in j is given by

$$L_j = \left(\frac{A_j \beta}{w_j}\right)^{\frac{1}{1-\beta-\alpha}} T_j^b. \tag{6}$$

We assume that $1-\beta > \alpha$ to guarantee that local labor demand is downward sloping, and that business land is owned by absentee landlords. This assumption means that the congestion force, determined by $(1-\beta)$, is stronger than the agglomeration force, determined by α.

1.2 Individuals

In each location j of the city, the residential area is composed of a continuum of residents who work in the business areas of different locations i. They experience their place of residence differently depending on local residential amenities that, in turn, depend on the number of residents who live in the same area. We assume that amenities in the residential area of location j are given by an increasing function of the number of residents in j, R_j. In addition, residents differ in their preferences for where to work according to a random idiosyncratic component s. This component captures the idea that individuals residing in a particular location j can have idiosyncratic reasons for working in different locations of the city.

We model the idiosyncratic preference component and amenities features associated with residing in location j, and working in location i, as scaling the utility of region$-j$

residents by $sB\left(R_j; j\right)$. We assume that s is drawn from a Fréchet distribution specific to commuting from the residential area of location j to the business area of location i,

$$Pr(s_{ij} \leq s) = e^{-\lambda_{ij} s^{-\theta}}, \ \lambda_{ij} > 0, \ \theta > 0. \tag{7}$$

Thus, residents from j who commute to i, with idiosyncratic preference s for that location, consume goods, $C_{ij}(s)$, and housing, $H_{ij}(s)$, from which they derive utility $sB(R_j; j)$ $C_{ij}(s)^{\gamma} H_{ij}(s)^{1-\gamma}$, $\gamma \in (0, 1)$. As discussed in the main text, as a benchmark we let $B(R_j; j) = R_j^{\sigma_j}$, but we also explore an alternative case in which $B(R_j; j) = B_j R_j^{\sigma}$. In this appendix, we describe the model using both of these measures of amenities. We assume that $\sigma_j > 1 - \gamma$ or $\sigma > 1 - \gamma$, meaning that the congestion force embedded in rising housing demand as the number of residents increases is dominated by the externality in amenities. The scale parameter λ_{ij} in (7) determines the average utility from working in i when commuting from j and the shape parameter θ governs the dispersion of idiosyncratic utility.

Residents of j who work in i incur an associated iceberg commuting cost, $\kappa_{ij} \in [1, \infty)$, in utility terms. Conditional on living in j and working in i, the problem of a resident having drawn idiosyncratic utility component s is then given by

$$U_{ij}(s) = \max_{C_{ij}, H_{ij}} \frac{sB\left(R_j; j\right)}{\kappa_{ij}} \left(\frac{C_{ij}(s)}{\gamma}\right)^{\gamma} \left(\frac{H_{ij}(s)}{1-\gamma}\right)^{1-\gamma} \tag{8}$$
$$\text{subject to } w_i = q_j^r H_{ij}(s) + C_{ij}(s),$$

where q_j^r is the price of a unit of housing services in j. Hence,

$$\begin{aligned} C_{ij}(s) &= \gamma w_i, \\ H_{ij}(s) &= \frac{(1-\gamma)\, w_i}{q_j^r}, \end{aligned} \tag{9}$$

and

$$U_{ij}(s) = \frac{sB(R_j; j)}{\kappa_{ij}} \left[w_i\right]^{\gamma} \left[\frac{w_i}{q_j^r}\right]^{1-\gamma} = \frac{sB(R_j; j) w_i \left(q_j^r\right)^{\gamma-1}}{\kappa_{ij}}.$$

1.2.1 Distribution of Utility

Since residents of j who work in i have different preferences s for that location drawn from (7), it follows that

$$G_{ij}(u) = Pr\left(U_{ij} < u\right) = Z_{ij}\left(\frac{u\kappa_{ij}\left(q_j^r\right)^{1-\gamma}}{B(R_j; j) w_i}\right),$$

or

$$G_{ij}(u) = e^{-\Phi_{ij}u^{-\theta}},\ \Phi_{ij} = \lambda_{ij} \left(\frac{\left(q_j^r\right)^{1-\gamma}}{B(R_j;j)} \right)^{-\theta} \left(\frac{w_i}{\kappa_{ij}} \right)^{\theta}. \tag{10}$$

Each resident of j chooses to commute to the business area i that offers maximum utility among all business areas. Then,

$$\begin{aligned} G_j(u) &= \Pr(\max_i \{U_{ij}\} < u) = \prod_{i=1}^{I} \Pr\left(U_{ij} < u\right) \\ &= \prod_{i=1}^{I} e^{-\Phi_{ij}u^{-\theta}}. \end{aligned}$$

Therefore, we have that

$$G_j(u) = e^{-\Phi_j u^{-\theta}}, \Phi_j = \sum_{i=1}^{I} \Phi_{ij}. \tag{11}$$

Given this Frechét distribution for the utility associated with residential area j, the expected utility from residing in j is

$$U_j = \int_0^\infty \theta \Phi_j u^{-\theta} e^{-\Phi_j u^{-\theta}} du.$$

Consider the change of variables,

$$y = \Phi_j u^{-\theta}, dy = -\theta \Phi_j u^{-(\theta+1)} du.$$

Then,

$$U_j = \int_0^\infty \Phi_j^{\frac{1}{\theta}} y^{\frac{-1}{\theta}} e^{-y} dy,$$

which can in turn be written as

$$U_j = \Gamma \left(\frac{\theta - 1}{\theta} \right) \Phi_j^{\frac{1}{\theta}}.$$

The expected utility from living in j, therefore, is a weighted sum of the utilities gained from commuting to the different business areas (raised to the θ). Namely,

$$U_j = \Gamma \left(\frac{\theta - 1}{\theta} \right) B(R_j;j) \left(q_j^r\right)^{\gamma-1} \left(\sum_{i=1}^{I} \lambda_{ij} \left(\frac{w_i}{\kappa_{ij}} \right)^{\theta} \right)^{\frac{1}{\theta}},$$

where Γ is the gamma function.

1.2.2 Commuting Patterns

Let π_{ij} represent the proportion of residents living in j and commuting to i. Note that $\sum_{i=1}^{I} \pi_{ij} = 1$ for $j = 1, ..., \underline{J}$. Commuting patterns can then be described by the following relationships,

$$R_{ij} = \pi_{ij} R_j,$$

where R_{ij} is the number of residents in j commuting to the business area of location i, and

$$\pi_{ij} = \Pr\left[U_{ij} > \max_{n \neq i} \left\{U_{nj}\right\}\right].$$

From (10), we have that $G_{ij}(u) = e^{-\Phi_{ij}u^{-\theta}}$ so that $g_{ij}(u) = \theta u^{-(\theta+1)} \Phi_{ij} e^{-\Phi_{ij}u^{-\theta}}$. Therefore,

$$\pi_{ij} = \int_0^\infty \widetilde{G}_j(u) \theta u^{-(\theta+1)} \Phi_{ij} e^{-\Phi_{ij}u^{-\theta}} du,$$

where $\widetilde{G}_j(u)$ is defined analogously to that in equation (11) but with $\widetilde{\Phi}_j = \sum\limits_{n \neq i} \Phi_{nj}$, so that $\Phi_j = \widetilde{\Phi}_j + \Phi_{ij}$. It follows that

$$\begin{aligned} \pi_{ij} &= \int_0^\infty \theta u^{-(\theta+1)} \Phi_{ij} e^{-\Phi_{ij}u^{-\theta}} e^{-\widetilde{\Phi}_j u^{-\theta}} du \\ &= \int_0^\infty \theta u^{-(\theta+1)} \Phi_{ij} e^{-\Phi_j u^{-\theta}} du. \end{aligned}$$

Define the change of variables,

$$y = \Phi_j u^{-\theta}, \, dy = -\theta \Phi_j u^{-(\theta+1)} du.$$

Then, we have that

$$\begin{aligned} \pi_{ij} &= \Phi_{ij} \int_0^\infty \theta u^{-(\theta+1)} e^{-\Phi_j u^{-\theta}} du \\ &= \frac{\Phi_{ij}}{\Phi_j} \int_0^\infty e^{-y} dy \\ &= \frac{\Phi_{ij}}{\Phi_j}, \end{aligned}$$

where $\Phi_{ij} = \lambda_{ij} \left(\frac{\left(q_j^r\right)^{1-\gamma}}{B(R_j;j)} \right)^{-\theta} \left(w_i/\kappa_{ij}\right)^\theta$ and $\Phi_j = \left(B(R_j;j) \left(q_j^r\right)^{\gamma-1} \right)^\theta \sum\limits_{i=1}^{I} \lambda_{ij} (w_i/\kappa_{ij})^\theta$. Therefore,

$$\pi_{ij} = \frac{\lambda_{ij} \left(w_i/\kappa_{ij}\right)^\theta}{\sum\limits_{i=1}^{I} \lambda_{ij} (w_i/\kappa_{ij})^\theta}. \tag{12}$$

The proportion of those living in j who commute to i for work depends on wages earned in i, net of commuting, relative to average net wages from commuting elsewhere (raised to the θ). Amenities by way of R_j do not affect commuting patterns from j to i since all residents of j experience the same externality equally.

1.2.3 The Residential Market

Recall that $H_{ij}(s) = H_{ij}$ denotes housing consumption for those living in j and commuting to i. It follows that average housing per resident in area j, H_j is given by

$$\begin{aligned}
H_j &= \sum_{i=1}^{I} \pi_{ij} H_{ij} \\
&= \frac{(1-\gamma)}{q_j^r} \sum_{i=1}^{I} \pi_{ij} w_i \\
&= \frac{(1-\gamma)}{q_j^r} \sum_{i=1}^{I} \frac{\lambda_{ij} \left(w_i / \kappa_{ij} \right)^{\theta}}{\sum_{n=1}^{I} \lambda_{nj} (w_n / \kappa_{nj})^{\theta}} w_i \\
&= \frac{(1-\gamma)}{q_j^r \sum_{n=1}^{I} \lambda_{nj} (w_n / \kappa_{nj})^{\theta}} \sum_{i=1}^{I} \lambda_{ij} \kappa_{ij}^{-\theta} w_i^{1+\theta}.
\end{aligned}$$

Since T_j^r denotes the total number of units of developed residential land in location j, equilibrium in residential market j must satisfy $R_j H_j = T_j^r$ or

$$R_j = \frac{q_j^r T_j^r}{(1-\gamma)\overline{w}_j} \tag{13}$$

where $\overline{w}_j$ is the average wage of residents of j:

$$\overline{w}_j = \sum_{i=1}^{I} \pi_{ij} w_i. \tag{14}$$

The numerator of (13) represents the value of developed land while the denominator captures income spent on housing per resident working across all business areas. It follows that

$$q_j^r = \frac{(1-\gamma) R_j \overline{w}_j}{T_j^r}. \tag{15}$$

This expression for the residential bid-rent increases with the average wage in location j.

If $\bar{u}$ is the utility available to agents in equilibrium in alternative cities, then populated residential areas must be such that $U_j = \Gamma\left(\frac{\theta-1}{\theta}\right)\Phi_j^{\frac{1}{\theta}} \geq \bar{u}$ or

$$B(R_j;j)\left(q_j^r\right)^{\gamma-1}\left[\sum_{i=1}^{I}\lambda_{ij}\left(\frac{w_i}{\kappa_{ij}}\right)^{\theta}\right]^{\frac{1}{\theta}} \geq \frac{\bar{u}}{\Gamma\left(\frac{\theta-1}{\theta}\right)}.$$

Substituting from (15), this last expression may equivalently be written as

$$B(R_j;j)R_j^{\gamma-1}\left(T_j^r\right)^{1-\gamma}(1-\gamma)^{\gamma-1}\overline{w}_j^{\gamma-1}\left[\sum_{i=1}^{I}\lambda_{ij}\left(\frac{w_i}{\kappa_{ij}}\right)^{\theta}\right]^{\frac{1}{\theta}} \geq \frac{\bar{u}}{\Gamma\left(\frac{\theta-1}{\theta}\right)}.$$

Therefore, to be viable, the area needs to have at least

$$R_j \geq \left(\frac{\bar{u}\,(1-\gamma)^{1-\gamma}\,\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left(T_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}}\right)^{\frac{1}{\sigma_j+\gamma-1}} \tag{16}$$

if $B(R_j;j) = R_j^{\sigma_j}$, or

$$R_j \geq \left(\frac{\bar{u}\,(1-\gamma)^{1-\gamma}\,\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)B_j\left(T_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}}\right)^{\frac{1}{\sigma+\gamma-1}} \tag{17}$$

if $B(R_j;j) = B_jR_j^{\sigma}$.

1.2.4 The City Labor Market

Since $\pi_{ij}R_j$ denotes the number of residents living in j who commute to the business area of i for work, we have that

$$l_iT_i^b = \sum_{j=1}^{\underline{J}}\pi_{ij}R_j$$

where $\underline{J} \leq J$ denotes areas of the city inhabited by residents or, equivalently,

$$\left(\frac{A_i\beta}{w_i}\right)^{\frac{1}{1-\beta-\alpha}}T_i^b = \sum_{j=1}^{\underline{J}}\pi_{ij}R_j. \tag{18}$$

In this case, there are $J - \underline{J}$ vacant residential areas in the city.[1] This last equation can be equivalently written as

$$\left(\frac{A_i\beta}{w_i}\right)^{\frac{1}{1-\beta-\alpha}} T_i^b = \sum_{j=1}^{\underline{J}} \pi_{ij} \left(\frac{\bar{u}\left(1-\gamma\right)^{1-\gamma} \overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left(T_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I} \lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma_j+\gamma-1}} \tag{19}$$

if $B(R_j; j) = R_j^{\sigma_j}$, or

$$\left(\frac{A_i\beta}{w_i}\right)^{\frac{1}{1-\beta-\alpha}} T_i^b = \sum_{j=1}^{\underline{J}} \pi_{ij} \left(\frac{\bar{u}\left(1-\gamma\right)^{1-\gamma} \overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right) B_j \left(T_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I} \lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma+\gamma-1}} \tag{20}$$

if $B(R_j; j) = B_j R_j^{\sigma}$. Given T_j^r, this is a system of I equations in I unknowns, w_i, $i = 1, ..., I$. Some locations may be purely commercial and thus may not have a residential district, either because the location is not zoned for residential use or no one has chosen to live there, or vice versa and instead be purely residential with no commercial district, so that in general, it is the case that $I \neq J$ and $I \neq \underline{J}$.

1.3 Residential Developers

There is a large number of small developers, none of whom can internalize residential externalities. Let h_j denote the number of units of land developed by a residential developer. Suppose further that the cost associated with developing h_j units of land is given by a convex variable cost, $V\left(h_j\right)$, that we specify as $V\left(h_j\right) = V h_j^{v}$ for $v > 1$, in addition to a fixed cost, $F_j > 0$. Residential developers then maximize

$$\max_{h_j} \Pi_j = h_j q_j^r - V\left(h_j\right) - F_j = \max_{h_j} h_j q_j^r - V h_j^{v} - F_j, \tag{21}$$

so that, if $h_j > 0$,

$$q_j^r = vVh_j^{v-1} \Leftrightarrow h_j = \left(\frac{q_j^r}{vV}\right)^{\frac{1}{v-1}} = \left(\frac{(1-\gamma)R_j\overline{w}_j}{vVT_j^r}\right)^{\frac{1}{v-1}}. \tag{22}$$

[1]Note that labor demand on the LHS is standard and decreasing in w_i. Labor supply on the RHS, however, is more difficult to characterize since it depends on the number of developed tracts, $\underline{J}$, which itself depends on the distribution of wages in equilibrium.

Furthermore, developers enter as long as profits are non-negative, so that $h_j > 0$ if $\Pi_j \geq 0$, or alternatively,

$$h_j q_j^r \geq V h_j^v + F_j,$$

in which case

$$(v-1)\, V h_j^v \geq F_j.$$

Alternatively, $\Pi_j \geq 0$ implies that

$$(v-1)\, V \left(\frac{(1-\gamma) R_j \overline{w}_j}{v V T_j^r}\right)^{\frac{v}{v-1}} \geq F_j, \tag{23}$$

so that

$$R_j \geq \frac{v V \left(\frac{F_j}{(v-1)V}\right)^{\frac{v-1}{v}}}{(1-\gamma)\, \overline{w}_j} T_j^r. \tag{24}$$

Hence, an individual developer will only invest in developing residential land if the density of residents R_j / T_j^r, which determines residential prices, is large enough.

Clearly, summing over developers, $n_j h_j = R_j H_j = T_j^r \leq \overline{T}_j^r$, and so

$$n_j = \frac{\left(T_j^r\right)^{\frac{v}{v-1}}}{\left(\frac{(1-\gamma)}{vV} R_j \overline{w}_j\right)^{\frac{1}{v-1}}}$$

if (24) is satisfied and $n_j = 0$ otherwise.

1.4 The City Equilibrium

Suppose that area j is inhabited by residents who commute to various locations in the city to work. Let Ω^F represent the set of locations where the amount of developed land has exhausted the amount of available buildable land, $\Omega^F = \{j | T_j^r = \overline{T}_j^r\}$. Let Ω^S denote the set of locations with residential development where the zoning constraint is not binding, $\Omega^S = \{j | 0 < T_j^r < \overline{T}_j^r\}$. We refer to these locations as semi-developed or partially developed. In all locations, wages satisfy equation (18). As the city responds to exogenous forces, locations may become fully developed, switching from Ω^S to Ω^F in equilibrium, or instead steadily depreciate with some lots eventually becoming vacant or empty parcels, switching from Ω^F to Ω^S.

1.4.1 The Case of Fully Developed Areas

From equation (15) or (22), in equilibrium, we have that for all locations $j \in \Omega^F$,

$$q_j^r = (1-\gamma)\frac{R_j}{\overline{T}_j^r}\overline{w}_j \tag{25}$$

Furthermore, residents can move freely across different areas of the city so that

$$R_j = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left(\overline{T}_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma_j+\gamma-1}} \tag{26}$$

if $B(R_j;j) = R_j^{\sigma_j}$, or

$$R_j = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)B_j\left(\overline{T}_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma+\gamma-1}}. \tag{27}$$

if $B(R_j;j) = B_j R_j^{\sigma}$.

In these locations, from (23), since all available land for potential development is already built up, developers may earn strictly positive rents,

$$(v-1)\,V\left(\frac{(1-\gamma)R_j\overline{w}_j}{vV\overline{T}_j^r}\right)^{\frac{v}{v-1}} > F_j,$$

in which case it follows that

$$R_j > \frac{vV\left(\frac{F_j}{(v-1)V}\right)^{\frac{v-1}{v}}}{(1-\gamma)\,\overline{w}_j}T_j^r.$$

1.4.2 The Case of Partially Developed Areas

For all other locations $j \in \Omega^S$, the fact that $T_j^r < \overline{T}_j^r$ under free entry in the market for land development implies, from (23), that in equilibrium,

$$R_j = \frac{vV\left(\frac{F_j}{(v-1)V}\right)^{\frac{v-1}{v}}}{(1-\gamma)\,\overline{w}_j}T_j^r. \tag{28}$$

In that case, free mobility of residents across areas of the city implies that

$$R_j = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left(T_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma_j+\gamma-1}}, \tag{29}$$

if $B(R_j; j) = R_j^{\sigma_j}$, or

$$R_j = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)B_j\left(T_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma+\gamma-1}}, \tag{30}$$

if $B(R_j; j) = B_j R_j^{\sigma}$. In the former case, solving for T_j^r from equations (28) and (29) gives

$$T_j^r = \overline{w}_j \left(\frac{\overline{u}(1-\gamma)^{1-\gamma}\left[\left(\frac{(v-1)V}{F_j}\right)^{\frac{v-1}{v}}\left(\frac{1-\gamma}{vV}\right)\right]^{\sigma+\gamma-1}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left\{\sum_{i=1}^{I}\lambda_{ij}\left[w_i/\kappa_{ij}\right]^{\theta}\right\}^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma_j}}. \tag{31}$$

In the latter case, solving for T_j^r from equations (28) and (30)

$$T_j^r = \overline{w}_j \left(\frac{\overline{u}(1-\gamma)^{1-\gamma}\left[\left(\frac{(v-1)V}{F_j}\right)^{\frac{v-1}{v}}\left(\frac{1-\gamma}{vV}\right)\right]^{\sigma+\gamma-1}}{B_j\Gamma\left(\frac{\theta-1}{\theta}\right)\left\{\sum_{i=1}^{I}\lambda_{ij}\left[w_i/\kappa_{ij}\right]^{\theta}\right\}^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma}}, \tag{32}$$

where $T_j^r < \overline{T}_j^r$ in both cases.[2]

The equilibrium in a particular area is determined by the crossing of (28) and (29) or (30). There are two potential equilibria, one with positive development and one without development. Both equilibria always exist, independent of other parameter values. In

[2]We conjecture that developed land is increasing in $\bar{u}$ but decreasing in w_i and F_j. Similarly, R_j is increasing in $\bar{u}$ but decreasing in w_i. In contrast, R_j is increasing in F_j since $\gamma < 1$.

order for an area to coordinate on the equilibrium with positive development, enough developers have to expect that others will invest, so that (28) is satisfied.

With identical developers, both types of expectations (development or no development) are rational and self-fulfilling. Clearly, if all residential areas coordinate in the bad equilibrium, the city disappears. However, as more areas coordinate in the bad equilibrium, the wage increases and the residents needed to make the area suitable for development decrease.

In this case, equations (15) and (28) may be combined to give a simple expression for (per unit) residential prices:

$$q_j^r = \nu V \left(\frac{F_j}{(v-1)\,V} \right)^{\frac{v-1}{v}}.$$

2 Solving the Model

The parameters of the model are: $\mathcal{P} = (v, V, \alpha, \beta, \gamma, \kappa_{ij}, \lambda_{ij}, \theta, A_i, F_j, \overline{T}_j^r, \sigma_j, \overline{u})$.

Solving the model given $\mathcal{P}$, or carrying out counterfactual exercises given an alternate set of parameters $\mathcal{P}'$, may be summarized as follows.

We have from equation (12)

$$\pi_{ij} = \frac{\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}}{\sum_{i=1}^{I} \lambda_{ij}(w_i/\kappa_{ij})^{\theta}}. \tag{33}$$

Then, from equations (31) and (32), we have

$$T_j^r = \begin{cases} \overline{T}_j^r & \text{if } j \in \Omega^F \\ \frac{(1-\gamma)R_j\overline{w}_j}{q_j^r}, & \text{if } j \in \Omega^S \end{cases}, \tag{34}$$

and from equations (29) and (30),

$$R_j = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left(T_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma_j+\gamma-1}} \tag{35}$$

if $B(R_j; j) = R_j^{\sigma_j}$, or

$$R_j = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma} \overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right) B_j \left(T_j^r\right)^{1-\gamma} \left[\sum_{i=1}^{I} \lambda_{ij} \left(w_i / \kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma+\gamma-1}} \tag{36}$$

if $B(R_j; j) = B_j R_j^{\sigma}$.

Residential prices are given by

$$q_j^r = \begin{cases} (1-\gamma)\overline{w}_j \frac{R_j}{\overline{T}_j^r} & \text{if } j \in \Omega^F \\ \left(\frac{F_j}{(v-1)V}\right)^{\frac{v-1}{v}} vV & \text{if } j \in \Omega^S \end{cases}. \tag{37}$$

Finally from equation (18), reproduced below, wages solve

$$\left(\frac{A_i \beta}{w_i}\right)^{\frac{1}{1-\beta-\alpha}} T_i^b = \sum_{j=1}^{J} \pi_{ij} R_j. \tag{38}$$

In general, equations (33) through (38) make up a system of $J^2 + 4J$ equations in the same number of unknowns given by $\pi_{ij}(\mathcal{P})$, $T_j^r(\mathcal{P})$, $R_j(\mathcal{P})$, $q_j^r(\mathcal{P})$ and $w_i(\mathcal{P})$. With $J = 1151$ census tracts as in our application, $J^2 + 4J = 1,329,405$.

In solving this system, we must be cognizant that the set of fully developed locations, Ω^F, and the set of semi-developed locations, Ω^S, are themselves endogenous and determined as part of the equilibrium. In other words, given that

$$T_j^r(R_j, w_j, \pi_{ij}, q_j^r; \mathcal{P}) = \frac{(1-\gamma) R_j \overline{w}_j}{q_j^r}$$

in the lower portion of equation (34) represents an unconstrained level of development for census tract j, it must be the case that

$$j \in \begin{cases} \Omega^F & \text{if } T_j^r(R_j, w_j, \pi_{ij}, q_j^r; \mathcal{P}) \geq \overline{T}_j^r \\ \Omega^S & \text{if } T_j^r(R_j, w_j, \pi_{ij}, q_j^r; \mathcal{P}) < \overline{T}_j^r \end{cases}. \tag{39}$$

This implies that the sets of fully developed and partially developed locations Ω^F and Ω^S potentially change in counterfactual scenarios relative to a given benchmark. All other equilibrium allocations follow in a straightforward manner.

3 Model Inversion and Calibration

We use the currently observed spatial allocations of Detroit to establish a benchmark for the quantitative spatial framework described above. In particular, we now describe how this framework can be used to rationalized available data on Detroit.

3.1 Vacant, Partially Developed, and Fully Developed Locations

Recall that in the model, J denotes the number of locations considered in our study, $\underline{J}$ the number of locations with residential population, and I the number of locations with active business districts. Our unit of analysis throughout the paper is a census tract, the smallest geographical unit for which we have a nearly complete matching dataset for the various key variables in the model.

Using the thresholds for classifying vacant census tracts described in the main text, we find that 52 of the 297 census tracts in Detroit proper are associated with the no-development case. The surrounding metro area (Wayne County, Oakland County, Macomb County) includes 866 additional tracts. Of the 1163 total tracts, 12 are omitted from the analysis because of measurement issues. Thus, in the end we have that $J = 1151$ and $\underline{J} = 1099$. In the 1151 census tracts we consider, all but one tract contain some measure of business activity (i.e. there is only one purely residential tract with no measured employment), so that $I = 1150$.

Residential population, R_j, and residential prices, q_j^r, are set to 0 in the 52 vacant tracts. Equations (29) and (30) then imply that total developed residential land, T_j^r, equals 0 in vacant tracts as well. In a given counterfactual, any combination of these tracts can be opened up for development. The assignment of parameter values to vacant tracts, whether they remain vacant or are opened up for development in a given counterfactual, is described throughout this section.

Of the census tracts not considered vacant, the model distinguishes between tracts that are fully developed, $\Omega^F = \left\{ j | T_j^r = \bar{T}_j^r \right\}$, and those that are partially developed, $\Omega^S = \left\{ j | 0 < T_j^r < \bar{T}_j^r \right\}$. Because the extent to which tracts are available to be developed, the percentage of empty parcels, is only available for the 297 census tracts within Detroit proper, we must impute this allotment for the remaining census tracts of Macomb, Oakland, and Wayne Counties. To do so, we estimate the following relationship using the data available on Detroit proper:

$$\ln \%Vacant_j = \omega_1 + \omega_2 R_j + \omega_3 T_j^r + \omega_4 \overline{w}_j + \omega_5 \kappa_{1j} + \epsilon_j,$$

where $\%Vacant_j$ is the ratio of vacant parcels and parcels with unoccupied structures to the total number of parcels, and κ_{1j} denotes the distance from j to the CBD (labeled as tract 1 in our analysis).

Table 1

	(1) $\ln(\%Vacant_j)$
ω_1	6.26
	(0.95)***
R_j	-0.0012
	(0.0002)***
T_j^r	-0.38
	(0.20)
$\overline{w}_j$	-0.00015
	(0.000024)***
κ_{1j}	-0.06
	(0.0081)***
R-squared	0.59
Observations	242

Given the findings in Table 1, we impute the percent of vacant parcels in the census tracts of the larger metro area surrounding Detroit. As a benchmark, we then designate a tract where more than 2/3 of its parcels are occupied as being fully developed. Other tracts with less than 2/3 occupancy are considered only partially developed.

3.2 Location-Specific Parameters

The parameters $(\lambda_{ij}, A_j, F_j, \overline{T}_j^r, \sigma_j, B_j)$ are high dimensional objects that cannot be easily estimated. We choose these parameters so that, conditional on $(v, V, \alpha, \beta, \gamma, \theta, \overline{u}, \sigma)$, equilibrium benchmark allocations are consistent with the data $(\pi_{ij}, w_j, q_j^r, R_j)$.

Given data on residential prices, q_j^r, and residential population, R_j, we can recover either the development costs or the upper bound on land development depending on whether a tract is considered partially or fully developed. In particular, note that in the benchmark case, our model necessitates recovering one parameter or the other for a given census tract but not both. Thus, from equation (37), we have that, for full census tracts,

$$\overline{T}_j^r = \frac{(1-\gamma)R_j\overline{w}_j}{q_j^r} \text{ if } j \in \Omega^F. \tag{40}$$

We assign the upper bound on land development to partially developed census tracts using

$$\overline{T}_j^r = T_j^A(1+x) \text{ if } j \in \Omega^S, \tag{41}$$

where T_j^A is total land suitable for residential purposes (including both developed and undeveloped land) and x is the maximum ratio of living to land area within the tract and the nearest three tracts.

Again from equation (37), we have that, for partially developed tracts,

$$F_j = (\nu - 1)V\left(\frac{q_j^r}{\nu V}\right)^{\frac{\nu}{\nu-1}} \text{ if } j \in \Omega^S. \tag{42}$$

Fully developed census tracts are assigned the median F_j value of the partially developed census tracts. If a fully developed census tract's q_j^r value with this median F_j value using equation (37) is higher than its observed value, we assign it an F_j value using equation (42) again. Since residential prices are set to zero in empty census tracts, $F_j = 0$ for vacant tracts, while newly opened census tracts in a counterfactual are assigned the F_j value of the nearest partially developed census tract.

The parameters λ_{ij} may be recovered by way of the gravity equation. From equation (12),

$$\pi_{ij} = \frac{\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}}{\sum_{n=1}^{I}\lambda_{nj}(w_n/\kappa_{nj})^{\theta}},$$

where $\kappa_{ij} \in [1, \infty)$. If $\pi_{ij} = 0$, then either $\lambda_{ij} = 0$ or $\kappa_{ij} \to \infty$. Commuting patterns can be alternatively expressed in terms of the Head and Ries (2001) index,

$$\frac{\pi_{ij}}{\pi_{jj}} = \frac{\lambda_{ij}(w_i/\kappa_{ij})^{\theta}}{\lambda_{jj}(w_j/\kappa_{jj})^{\theta}}.$$

Then given θ (whose calculation is described in the following section), we obtain λ_{ij} using

$$\lambda_{ij} = \pi_{ij}\left(\frac{w_j}{w_i}\right)^{\theta}\left(\frac{\kappa_{ij}}{\kappa_{\min}}\right)^{\theta}\left(\frac{\lambda_{jj}}{\pi_{jj}}\right), \tag{43}$$

where $\kappa_{\min} = \kappa_{jj} \quad \forall j$, and we normalize commuting costs so that $\left(\frac{w_j}{\kappa_{\min}}\right)^{\theta}\left(\frac{\lambda_{jj}}{\pi_{jj}}\right) = 1$. Equation (43) then becomes

$$\lambda_{ij} = \pi_{ij}\left(\frac{\kappa_{ij}}{w_i}\right)^{\theta}.$$

For the one census tract with no measured business activity, λ_{ij} values are set to 0. When opening up empty census tracts for development in a given counterfactual, we assign those tracts the λ_{ij} values of the nearest partially developed census tract.

Since κ_{ij} is directly estimated using commute costs data, this approach to obtaining λ_{ij} using (43) presumes that we are also able to exactly match wages, w_j, as part of the model inversion; we show below that this can indeed be done through the choice of location-specific productivities, A_j. [3]

Next, we need to ensure that the benchmark model is consistent with the spatial wage distribution observed in Detroit and surrounding areas. Specifically, to match the observed distribution of wages across census tracts in the benchmark scenario, we choose the productivity parameters, A_i, so that observed wages solve the labor market clearing equation (38),

$$A_i = \frac{w_i}{\beta}\left(\frac{1}{T_i^b}L_i\right)^{1-\beta-\alpha}.$$

Finally, if using the benchmark measure of amenities, $B(R_j;j) = R_j^{\sigma_j}$, then given equation (35), we solve for σ_j's, normalizing $\bar{u}$ to 1, that are consistent with the distribution of residential population across census tracts,

$$\sigma_j = \left(\frac{1}{\ln\left(R_j\right)}\right)\ln\left(\frac{\bar{u}\left(1-\gamma\right)^{1-\gamma}\left(\overline{w}_j\right)^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left(T_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}}\right)-\gamma+1,$$

where $T_j^r = (1-\gamma)R_j\overline{w}_j/q_j^r$, with q_j^r calculated based off of equation (37).

If instead using the alternative measure of amenities, $B(R_j;j) = B_jR_j^{\sigma}$, then given equation (36), we solve for B_j's, normalizing $\bar{u}$ to 1, that are consistent with the distribution of residential population across census tracts,

$$B_j = \frac{\bar{u}\left(1-\gamma\right)^{1-\gamma}\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)R_j^{\sigma+\gamma-1}\left(T_j^r\right)^{1-\gamma}\left[\sum_{i=1}^{I}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}},$$

[3]Consider the term $\left\{\sum_{i=1}^{I}\lambda_{ij}\left[w_i/\kappa_{ij}\right]^{\theta}\right\}^{\frac{1}{\theta}}$ in these equations. From equation (43), we have that $\lambda_{ij}\left(\frac{w_i}{\kappa_{ij}}\right)^{\theta} = \pi_{ij}\left(\frac{w_j}{w_i}\right)^{\theta}\left(\frac{\kappa_{ij}}{\kappa_{\min}}\right)^{\theta}\left(\frac{\lambda_{jj}}{\pi_{jj}}\right)\left(\frac{w_i}{\kappa_{ij}}\right)^{\theta} = \pi_{ij}\left(\frac{w_j}{\kappa_{\min}}\right)^{\theta}\left(\frac{\lambda_{jj}}{\pi_{jj}}\right)$. Then $\left\{\sum_{i=1}^{I}\lambda_{ij}\left(\frac{w_i}{\kappa_{ij}}\right)^{\theta}\right\}^{\frac{1}{\theta}} = \left\{\sum_{i=1}^{I}\pi_{ij}\left(\frac{w_j}{\kappa_{\min}}\right)^{\theta}\left(\frac{\lambda_{jj}}{\pi_{jj}}\right)\right\} = \left(\frac{w_j}{\kappa_{\min}}\right)\left(\frac{\lambda_{jj}}{\pi_{jj}}\right)^{\frac{1}{\theta}}$.

where again $T_j^r = (1-\gamma) R_j \overline{w}_j / q_j^r$.

3.3 Citywide Parameters

The values of the citywide parameters α, β, γ, and ν are set using standard sources in the literature. We estimate the parameters V and θ either because the literature does not offer a clear counterpart or because the context of Detroit is somewhat unique. The parameter V, which governs the level of the variable cost of residential construction, is estimated to match the mean of the distribution describing the number of contractors with active permits in the benchmark year (2014), across partially developed census tracts:

$$n_j = (\nu V)^{\frac{1}{\nu-1}} (q_j^r)^{\frac{\nu}{1-\nu}} (1-\gamma) R_j \overline{w}_j.$$

The mean of n_j in the data and implied by the model is 9.25 contractors with active permits per census tract. The standard deviation of n_j in the data is 4.52 and that implied by the model is 4.51 (not targeted). The data on the number of contractors comes from the Detroit Demolition Program and Buildings, Safety Engineering and Environmental Department.

The parameter θ governs the elasticity of commuting flows with respect to commuting costs. Similarly to Monte, Redding, and Rossi-Hansberg (2015), we estimate the following equation

$$\log\left(\frac{\pi_{ij}}{\pi_{jj}}\right) = -\theta \log\left(\frac{\kappa_{ij}}{\kappa_{jj}}\right) + \mu_i + \mu_j + u_{ij}, \tag{44}$$

which, given commute costs data (distance or time) and fixed origin and destination effects, μ_i and μ_j, allows us to estimate θ. Here we estimate θ using commuting costs based on all pairwise commute times between census tracts.

As discussed in the main text and the robustness appendix, when using the alternative measure of amenities, $B(R_j; j) = B_j R_j^{\sigma}$, we estimate σ two different ways: once using cross-sectional data and once using data on the change in amenities over time. In the first case, we estimate the following equation:

$$\ln\left(\overline{B}_j\right) = b + \sigma \ln\left(R_j\right) + \varphi X_j + e_j,$$

where X_j is a vector of tract controls consisting of each tract's distance to sets of various fixed amenities in the Greater Detroit area, and business productivity, A_i, is used as an instrument for R_j.

In the second case, we collect data on prices, wages, and residents for 2004 and then estimate the following equation:

$$\ln\left(\frac{\overline{B}_{j,t}}{\overline{B}_{j,t-1}}\right) = \tilde{b} + \sigma \ln\left(\frac{R_{j,t}}{R_{j,t-1}}\right) + \tilde{e}_j,$$

where t denotes 2014, our benchmark year, and $t-1$ denotes 2004. We use as instruments for changes in R_j the change in business productivity, A_i, over the period 2004-2014 and the log of the mean distance from each census tract to four census tracts containing auto plants that closed during the Great Recession

Values of the citywide parameters, along with their sources, are listed below.

α: 0.06 - Cicone and Hall, (1996), "Productivity and the Density of Economic Activity", American Economic Review, vol. 86, pp. 54-70

β: 0.8 - Ahlfeldt, Redding, Sturm, Wolf (2015), "The Economics of Density: Evidence from the Berlin Wall," Econometrica, vol. 83(6), pp. 2127-2189

γ: 0.76 - Davis, Ortalo-Magné, (2011), "Household Expenditures, Wages, Rents," Review of Economic Dynamics, vol. 14(2), pp. 248-261

θ: 8.34 - Gravity equation for commuting

ν: 2.5 - Ahlfeldt, McMillen, (2015), "The Vertical City: The Price of Land and the Height of Buildings in Chicago 1870-2010," SERC Discussion Paper 180

V: 175,472,386 - Equation for mean number of contractors

σ: 0.635 when using cross-sectional data; 0.519 when using data on the change in amenities over time

4 Solution Algorithm

This section describes the algorithm used to implement the model solution described previously.

1. Guess a vector of wages, $\mathbf{w}^0$.

2. Given this guess, calculate the matrix of commuting patterns,

$$\pi_{ij}(w_i = w_i^0) = \frac{\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}}{\sum_{i=1}^{I}\lambda_{ij}(w_i/\kappa_{ij})^{\theta}}.$$

3. Given $\mathbf{w}^0$ and step 2, from equations (31) and (32), calculate

$$T_j^r(w_i = w_i^0) = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\left[\left(\frac{(v-1)V}{F_j}\right)^{\frac{v-1}{v}}\left(\frac{1-\gamma}{vV}\right)\right]^{\sigma+\gamma-1}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left\{\sum_{i=1}^{I}\lambda_{ij}\left[w_i/\kappa_{ij}\right]^{\theta}\right\}^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma_j}} \overline{w}_j$$

if $B(R_j; j) = R_j^{\sigma_j}$, or

$$T_j^r(w_i = w_i^0) = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\left[\left(\frac{(v-1)V}{F_j}\right)^{\frac{v-1}{v}}\left(\frac{1-\gamma}{vV}\right)\right]^{\sigma+\gamma-1}}{\Gamma\left(\frac{\theta-1}{\theta}\right)B_j\left\{\sum_{i=1}^{I}\lambda_{ij}\left[w_i/\kappa_{ij}\right]^{\theta}\right\}^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma}} \overline{w}_j$$

if $B(R_j; j) = B_j R_j^{\sigma}$.

   (a) Check that $T_j^r(w_i = w_i^0)$ satisfies its upper bound constraint,

$$T_j^r(w_i = w_i^0) = \begin{cases} \overline{T}_j^r & \text{if } T_j^r(w_i = w_i^0) \geq \overline{T}_j^r \\ T_j^r(w_i = w_i^0) & \text{otherwise} \end{cases}$$

4. At guess $\mathbf{w}^0$, calculate residential population,

   (a) Open City case: City-wide utility, $\bar{u}$, is fixed and population $\sum_{j=1}^{J} R_j(w_i = w_i^0; \bar{u})$, is endogenous. Then

$$R_j(w_i = w_i^0) = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left(T_j^r(w_i = w_i^0)\right)^{1-\gamma}\left[\sum_{i=1}^{J}\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma_j+\gamma-1}} \tag{45}$$

if $B(R_j; j) = R_j^{\sigma_j}$, or

$$R_j(w_i = w_i^0) = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\overline{w}_j^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right) B_j \left(T_j^r(w_i = w_i^0)\right)^{1-\gamma} \left[\sum_{i=1}^{J} \lambda_{ij} \left(w_i/\kappa_{ij}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma+\gamma-1}} \tag{46}$$

if $B(R_j; j) = B_j R_j^{\sigma}$.

(b) Closed City case: Population is fixed and $\overline{u}$ is endogenous. Let P denote the fixed population of Greater Detroit in the benchmark scenario (where the population of vacant tracts is set to zero). This requires specifying a tolerance value Δ and using Newton's method to solve for $\overline{u}$ until $\sum_{i=1}^{J} R_j(w_i = w_i^0, \bar{u}) - P < \Delta$.

Note: Fix P and consider an alternative population, P'. Given $\mathbf{w}^0$, step 4b, delivers $\overline{u}$ lower than in the open city case in step 4a when $P' > P$. This property appears to persist as we iterate on wages, and thus all other variables, towards convergence in step 8 below.

5. At guess $\mathbf{w}^0$ in each census tract, calculate labor supply,

$$\sum_{j=1}^{J} \pi_{ij}(w_i = w_i^0) R_j(w_i = w_i^0), .$$

and labor demand,

$$L_i(w_i = w_i^0) = \left(\frac{A_i \beta}{w_i}\right)^{\frac{1}{1-\beta-\alpha}} T_i^b.$$

6. Calculate excess labor demand in each census tract,

$$X_i(w_i = w_i^0) = L_i(w_i = w_i^0) - \sum_{j=1}^{J} \pi_{ij}(w_i = w_i^0) R_j(w_i = w_i^0).$$

7. If $\sum_{i=1}^{I} |X_i(w_i = w_i^0)| \geq \varepsilon$, calculate a new wage vector, $\mathbf{w}^1$,

$$\mathbf{w}^1 = \mathbf{w}^0 + \delta \mathbf{X},$$

where $\mathbf{X} = (X_1, ..., X_I)'$.

Note: If using a constant wage adjustment factor δ, the solution algorithm can begin to slow down as excess labor supply/demand decreases. The algorithm can be sped up by increasing δ in census tracts where excess labor supply/demand is decreasing between iterations and decreasing δ in tracts where excess labor supply/demand is increasing between iterations.

8. Iterate on steps 1 through 8 until $\sum_{i=1}^{I} |X_i(w_i = w_i^0)| < \varepsilon$.

5 Policy Evaluation

Let Ω^v denote the set of 52 vacant tracts, and let $\Omega^P \subset \Omega^v$ denote the subset of vacant tracts opened up for development in a given counterfactual or policy experiment.

To persuade developers to coordinate their efforts, the city may use a combination of moral suasion and a land development guarantee in the amount of $\sum_{\Omega^P} \left(F_j + Vh_j^{'}\right)\left(n_j^{'} - 1\right)$, where $n_j^{'}$ and $h_j^{'}$ are new equilibrium values in the counterfactual policy experiment in the targeted vacant tracts ($n_j^{'}$ is rounded up to the nearest integer and $n_j^{'} - 1$ is set to equal 1 if $n_j^{'} = 1$). The cost of the policy has a lower bound of zero, if the development guarantee is successful in inducing coordination, and an upper bound of the full amount of the guarantee, $\sum_{\Omega^P} \left(F_j + Vh_j^{'}\right)\left(n_j^{'} - 1\right)$.

New residential rents in a counterfactual policy experiment are denoted by $q_j^{r'} T_j^{r'}$. Since residential rents can change across all census tracts in a counterfactual, the total change in city residential rents induced by the policy is $\sum_{i=1}^{J} \left(q_j^{r'} T_j^{r'} - q_j^r T_j^r\right)$. When the land guarantee solves the coordination problem among developers and residents, the dollar impact of the policy equals the total change in city residential rents. When the full land development guarantee ends up being used, the dollar impact of the policy is $\sum_{i=1}^{J} \left(q_j^{r'} T_j^{r'} - q_j^r T_j^r\right) - \sum_{\Omega^P} \left(F_j + Vh_j^{'}\right)\left(n_j^{'} - 1\right)$.